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OM

Sujal N. Patel

ECE

PM 1 (B).

ELECTROMAGNETIC THEORY.

(EMT)

Sir: Mr. Koteswara Rao.

Email. Id: nvkr1 @ rediffmail.com.

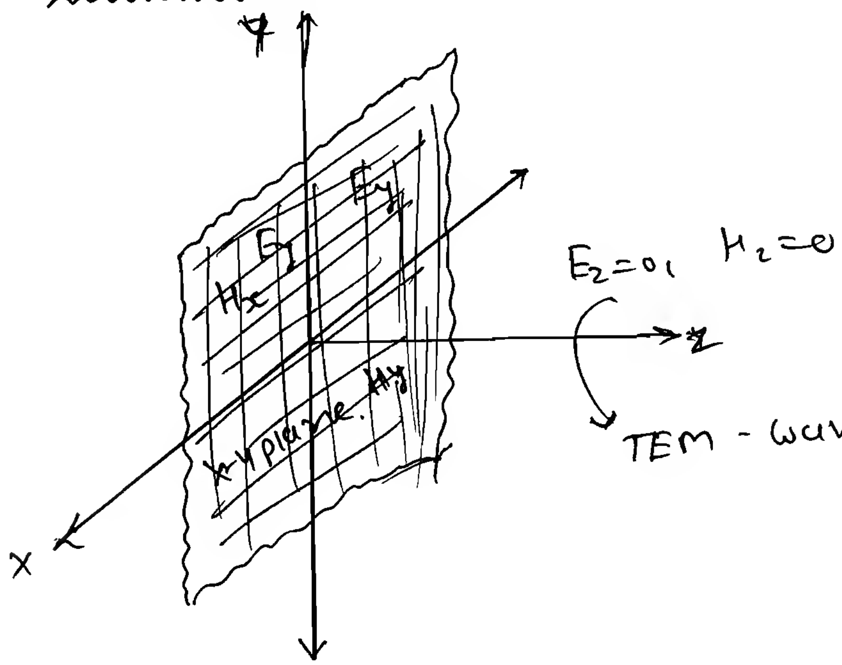
→ A constant can't be called as wave.

∴ We choose $\boxed{E_z = 0}$

→ When wave is propogating in z direction the wave do not have any field component along the direction of propogation i-e along z-direction.

→ The possible non-zero components are E_x, E_y, H_x, H_y.

* **Transverse Plane:-**



→ In general,

$$\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z .$$

$$\bar{H} = H_x \hat{a}_x + H_y \hat{a}_y + H_z \hat{a}_z .$$

⇒ Transverse plane:

→ It is that plane which is exist normal to the direction of propogation.

→ When a wave propogating along z-direction the transverse plane would be xy-plane.

→ When wave is propogating along z-direction the possible non-zero field components are E_x, E_y, H_x, & H_y lies in the x-y plane. i.e. in transverse plane.

→ The possible field components are lies in transverse plane and hence this wave is called transverse electromagnetic wave (or) TEM waves.

$$\frac{\partial^2}{\partial z^2}\left(E_x \hat{a}_x + E_y \hat{a}_y\right) = \mu\epsilon \frac{\partial^2}{\partial t^2}\left[E_x \hat{a}_x + E_y \hat{a}_y\right].$$

Compare $\hat{a}_x \Rightarrow$ $$\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}.$$

$\hat{a}_y \Rightarrow$ $$\frac{\partial^2 E_y}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_y}{\partial t^2}$$

similarly,

$$\frac{\partial^2 H_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 H_x}{\partial t^2}$$

$$\frac{\partial^2 H_y}{\partial z^2} = \mu\epsilon \frac{\partial^2 H_y}{\partial t^2}.$$

→ Scalar wave eqs 2nd order 2-dim PDEs.

→ Soln of these eqs are also said to be uniform plane waves.

→ Let us consider one of the equations.

$$\frac{\partial^2 E_y}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_y}{\partial t^2}.$$

→ By using a method of variable separable the function involved above can be thought of representing as a product of two independent functions:

$$E_y(z,t) \Rightarrow F_1(z) \cdot F_2(t) \quad e^{j\omega t}.$$

$F_1 \rightarrow$ Function of z - alone.

$F_2 \rightarrow$ Function of t - alone.

→ $E_y(z,t) = Re\left[E_s(z) \cdot e^{j\omega t}\right].$

→ Thus, we approximate the time variation is $e^{j\omega t}$,

$$\therefore \frac{\partial^2}{\partial z^2}\left[Re\left[E_{ys} \cdot e^{j\omega t}\right]\right] = \mu\epsilon \frac{\partial^2}{\partial t^2}\left\{Re\left(E_{ys}\, e^{j\omega t}\right)\right\}.$$

$$= +j^2\omega^2 \mu\epsilon\left\{Re\left[E_{ys}\, e^{j\omega t}\right]\right\}$$

$$\therefore \quad = -\omega^2\mu\epsilon\left\{Re\left[E_{ys}\, e^{j\omega t}\right]\right\}.$$

Suppress the time variation on both the sides.

$$\boxed{\frac{d^2 E_{ys}}{dz^2} + \omega^2\mu\epsilon\, E_{ys} = 0.}$$

This is a 2^{nd} order simple D.E.

→ The above eqn may be also called as harmonic eqn.

→ Solⁿ of harmonic eqⁿ may take either sine (or) cosine (or) exponent form. we consider exponent form.

$$\therefore \frac{d^2 E_{ys}}{dz^2} + \beta^2 E_{ys} = 0. \qquad \beta^2 = \omega^2 \mu \epsilon.$$

$$\beta = \omega\sqrt{\mu\epsilon}.$$

$$\therefore m^2 + \beta^2 = 0$$

$$\therefore m = \pm j\beta.$$

$$\therefore E_{ys} = C_1 e^{j\beta z} + C_2 e^{-j\beta z}.$$

$$\therefore E_y(z,t) = \mathrm{Re}\left[C_1 e^{-j\beta z} \cdot e^{j\omega t} + C_2 e^{j\beta z} \cdot e^{j\omega t}\right].$$

$$= \mathrm{Re}\left[\underbrace{C_1 e^{j(\omega t - \beta z)}}_{\text{Term-1}} + \underbrace{C_2 e^{j(\omega t + \beta z)}}_{\text{Term-2}}\right].$$

Term-1 → (+z) Term-2 → (−z).

⇒ Term-1

$$\underbrace{\boxed{C_1}}_{\text{Amplitude}} \cdot e^{j\overbrace{\boxed{(\omega t - \beta z)}}^{\text{Phase}}}$$

$$\therefore \omega t - \beta z = \text{Const.} \rightarrow (+z)$$

$$\therefore \omega dt - \beta dz = 0.$$

$$\therefore \frac{dz}{dt} = \omega/\beta.$$

$$\therefore \boxed{v_0 = \omega/\beta \ \text{m/s.}}$$

⇒ Term - ②

$$\boxed{C_2} \cdot e^{j\boxed{(\omega t + \beta z)}} \longrightarrow \text{phase.}$$

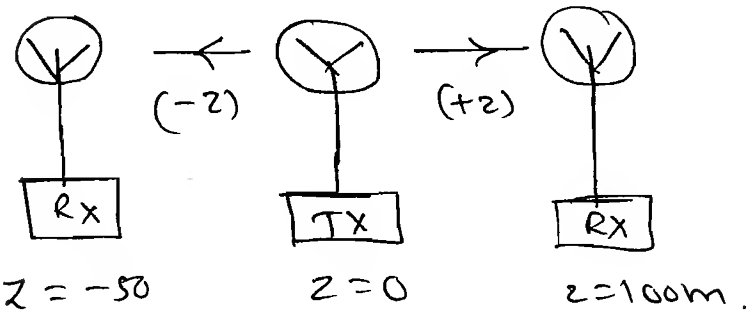
Amplitude

$\therefore \quad \omega t + \beta z = \text{Const.} \longrightarrow (-z).$

$\therefore \quad \omega\, dt + \beta\, dz = 0.$

$\therefore \quad \dfrac{dz}{dt} = -\omega/\beta.$

$\therefore \quad \boxed{v_0 = -\omega/\beta \ \ m/s.}$



Term-① in its mathamatical form indicates that the wave is propogating along +ve z-direction only where as Term-② in its mathamatical form indicates that the wave is propogating along -ve z-direction.

→ For the wave which are propogating along the +ve z direction we consider term-① only and we ignored term ②.

→ For the wave which are propogating along

$-ve$ z direction we considered term (2) only and we ignored term - (1).

→ The velocity of wave is $= \frac{\omega}{\beta}$.

* **Intrinsic Impedence: (η)**

→ Unit is 'Ω'.

- It is ratio of E to H.
- It will depends upon medium properties.

→ In a Linear Homogenius Isotropic Nonconducting medium this is given by,

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

In free space $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{10^{-9}}{36\pi}}}$

$$\therefore \eta = \sqrt{4 \times 36 \times \pi^2 \times 100}$$

$$\therefore \eta = 2 \times 6 \times \pi \times 10$$

$$\boxed{\eta = 120\pi}\ \Omega$$

→ $E_z = 0$, $H_z = 0$ → $(+z)$

$\partial/\partial x = 0$ $\partial/\partial y = 0$

$$\therefore \boxed{\frac{E_x}{E_y} = \boxed{\eta} = -\frac{E_y}{E_x}}.$$

- Impedence is not $-ve$, the ratio may be $-ve$.

$$\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + \underset{\shortparallel\, 0}{\boxed{E_z}} \hat{a}_z$$

$$\bar{H} = H_x \hat{a}_x + H_y \hat{a}_y + H_z \hat{a}_z \to 0$$

$$|\bar{E}| = \sqrt{E_x^2 + E_y^2} = \eta \sqrt{H_x^2 + H_y^2}$$

$$\therefore |\bar{H}| = \sqrt{H_x^2 + H_y^2}$$

$$\therefore \left|\frac{\bar{E}}{\bar{H}}\right| = \frac{\eta \sqrt{H_y^2 + H_x^2}}{\sqrt{H_x^2 + H_y^2}}$$

$$\therefore \left|\frac{\bar{E}}{\bar{H}}\right| = \eta .$$

$\rightarrow$ $\bar{E} \cdot \hat{a}_z = 0.$

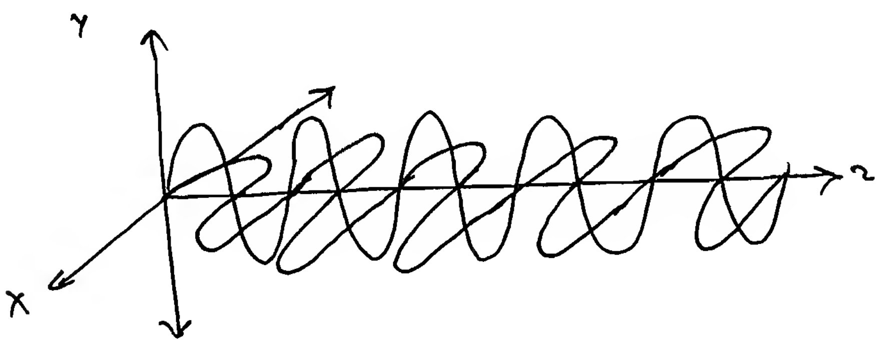
$\bar{H} \cdot \hat{a}_z = 0.$

$$\bar{E} \cdot \bar{H} = E_x H_x + E_y H_y.$$

$$= \eta H_y H_x - \eta H_x H_y .$$

$$\bar{E} \cdot \bar{H} = 0$$

$\rightarrow$ In a wave propugation, $\bar{E}$ & $\bar{H}$ and the vector corresponds to direction of propogation are mutually orthogonal to each other.



①

$\partial/\partial y() = 0, \ \partial/\partial z = 0$

$\xrightarrow[E_x = 0, \ H_x = 0]{} (+x)$

$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$

$\partial/\partial y() = 0, \ \partial/\partial z = 0$

$\xrightarrow[E_x = 0, \ H_x = 0]{} (-x)$

$\frac{E_y}{H_z} = -\eta = -\frac{E_z}{H_y}$

②

$\partial/\partial x = 0, \ \partial/\partial z = 0$

$\xrightarrow[H_y = 0, \ E_y = 0]{} (+y)$

$\frac{E_z}{H_x} = \eta = -\frac{E_x}{H_z}$

$\partial/\partial x = 0, \ \partial/\partial z = 0$

$\xrightarrow[H_y = 0, \ E_y = 0]{} (-y)$

$\frac{E_z}{H_x} = -\eta = -\frac{E_x}{H_z}$

③

$\partial/\partial x = 0, \ \partial/\partial y = 0$

$\xrightarrow[H_z = 0, \ E_z = 0]{} (+z)$

$\frac{E_x}{H_y} = -\eta = \frac{E_y}{H_x}$

$\partial/\partial x = 0, \ \partial/\partial y = 0$

$\xrightarrow[H_z = 0, \ E_z = 0]{} (-z)$

$\frac{E_x}{H_y} = -\eta = -\frac{E_y}{H_x}$

* $\beta = \omega\sqrt{\mu\epsilon}$ | $v_0 = \frac{1}{\sqrt{\mu\epsilon}}$

$\beta = \frac{\omega}{v}$

→ If 'f' is the freq of the wave and 'λ' is the corresponding wavelength, such that

$f\lambda = v_0$

'β' is phase shift constant

$\therefore \beta = \frac{2\pi f}{f\lambda} = \frac{2\pi}{\lambda}$ rad/m

'λ' ⟶ '2π' rad.

$\beta = \frac{2\pi}{\lambda}$ rad/m.

$\underline{\underline{Ex-1}}$ In air $\bar{E} = 50 \cos(10^8 t - \beta x)\, \hat{a}_y$ V/m where β is a +ve real. Find $\bar{H}$, β, direction propogation and also represents fields in phasor form.

Ans: In air $\Rightarrow$ μ_0 & ϵ_0. $\omega = 10^8$ rad/s

$$\therefore \quad \eta = \sqrt{\frac{\mu_0}{\epsilon_0}}.$$

$$\beta = \omega\sqrt{\mu_0 \epsilon_0}$$
$$= 10^8 / 3\times10^8$$
$$\boxed{\beta = 1/3}$$

$$\therefore \boxed{\eta = 120\pi}$$

$$\therefore \quad \bar{E} = 50 \cos(10^8 t - \beta x)\, \hat{a}_y$$

x-direction of propogation.

$$\overline{\qquad\qquad\qquad} +x$$

$$\boxed{\frac{E_y}{H_z} = +\eta} = -\frac{E_z}{H_y}.$$

because $(+\hat{a}_x)$.

$$\bar{H} = \frac{50}{\eta} \cos(10^8 t - \beta x)\, \hat{a}_z \rightarrow (\because \bar{E} \text{ is in } \hat{a}_y)$$

$$\therefore \boxed{\bar{H} = \frac{50}{120\pi} \cos\left(10^8 t - \frac{x}{3}\right) \hat{a}_z.}$$

$$\therefore \quad \bar{E}_s = 50 e^{-j\beta x}\, \hat{a}_y. \qquad \bar{H}_s = \frac{50}{\eta} e^{-j\beta x}\, \hat{a}_z$$

$$\therefore \quad \bar{E}_s = 50 e^{-jx/3}\, \hat{a}_y \qquad \bar{H}_s = \frac{50}{120\pi} e^{-j\frac{x}{3}}\, \hat{a}_z.$$

$$\left.\begin{aligned} &\bar{E}\cdot\hat{a}_x = 0 \\ &\bar{H}\cdot\hat{a}_x = 0 \\ &\bar{E}\cdot\bar{H} = 0 \end{aligned}\right\} \text{All are verified.}$$

Ex-2 In a non-magnetic medium. Let $\bar{H} = 10\cos(\omega t + 0.5z)\,\hat{a}_y$ A/m. Assume relative permitivity of medium is 4.0 find intrinsic impedence of the medium, freq. at which wave is propogating, β, direction propogation and also wright the expression for $\bar{E}$.

Ans:

Non magnetic medium =

$$\mu = \mu_0, \quad \epsilon = \epsilon_0 \epsilon_r.$$

$$\therefore \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{120\pi}{2}$$

$$\boxed{\eta = 60\pi\ \Omega}$$

$$\therefore \boxed{\beta = 0.5}$$

$$\therefore \beta = \omega\sqrt{\epsilon\mu}.$$

$$\therefore \beta = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

$$\omega = \frac{\beta}{\sqrt{\epsilon_r}\cdot\sqrt{\mu_0\epsilon_0}} = \frac{0.5 \times C}{\sqrt{\epsilon_r}}.$$

$$\boxed{C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \text{m/s}}$$

$$\therefore \omega = \frac{0.5 \times 10^8 \times \cancel{3}1.5}{2}$$

$$\therefore f = \frac{0.75 \times 10^8}{2\pi} \text{ Hz}$$

$$\therefore v_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{3\times10^8}{\sqrt{4}}$$

$$\therefore \boxed{v_0 = 1.5 \times 10^8 \text{ m/s.}}$$

$\longrightarrow (-z)$

$$\frac{E_x}{H_y} = -\eta = -\frac{E_y}{H_x}.$$

$\therefore \quad \bar{E} = -10\eta \cos(\omega t + 0.5z)\, \hat{a}_x.$

$\therefore$
$$\left.\begin{aligned} &\bar{E}\cdot\bar{H} = 0. \\ &\bar{E}\cdot(-\hat{a}_z) = 0. \\ &H\cdot(-\hat{a}_z) = 0 \end{aligned}\right\}$$ All are verified.

Ex-3 In a Lossless medium the intrinsic impedence is 250 Ω and given that
$\bar{E} = 20\cos(\omega t - \beta y)\,\hat{a}_x + 30\sin(\omega t - \beta z)\,\hat{a}_z$ V/m.
β is a positive real. Find direction of propogation and Expression for magnetic field intensity.

Ans:
$$\bar{E} = \underbrace{20\cos(\omega t - \beta y)}_{E_x}\,\hat{a}_x + \underbrace{30\sin(\omega t - \beta z)}_{E_z}.\,\hat{a}_z.$$
$+y.$

$\therefore \longrightarrow +y.$

$$\frac{E_z}{H_x} = +\eta = -\frac{E_x}{H_z}.$$

$$\therefore \quad H_x = \frac{E_z}{\eta}, \qquad H_z = -\frac{E_x}{\eta}.$$

$$\therefore \quad \bar{H} = \frac{30\sin(\omega t - \beta z)}{\eta}\,\hat{a}_x - \frac{20}{\eta}\cos(\omega t - \beta y)\,\hat{a}_z.$$

→ In the above problem write the Electric field quantity in the phasor form.

$$\bar{E} = 20\cos(\omega t - \beta y)\hat{a}_x + 30\cos(\omega t - \beta y - \tfrac{\pi}{2})\hat{a}_z \text{ V/m.}$$

$$\therefore \bar{E} = \mathrm{Re}\left[20 e^{j(\omega t - \beta y)} \cdot \hat{a}_x + 30 e^{j(\omega t - \beta y - \frac{\pi}{2})} \cdot \hat{a}_z\right].$$

$$\therefore \bar{E}_s = 20 e^{-j\beta y}\hat{a}_x + 30 e^{-j(\beta y + \frac{\pi}{2})}\hat{a}_z.$$

Ex-3 Let, $\bar{E} = 30\sin(10^8 t - \frac{x}{2})\hat{a}_x$ V/m. Find the direction of propagation and also find the expression for $\bar{H}$.

Ans The given $\bar{E}$ doesn't represent a valid EM wave.

* Wave Propagation in a Conducting unbounded medium (or) Lossy Charge free medium.

→ writing the Maxwell's eqn's for the above assummed medium.

→ Inside the Conductor the Charge is zero. and also we are assuming Charge free medium i.e. $\rho_v = 0$. indicates $\nabla \cdot \bar{E} = 0$.

① $\nabla \times \bar{E} = -\mu \dfrac{\partial \bar{H}}{\partial t}$.

② $\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \dfrac{\partial \bar{E}}{\partial t}$.

③ $\nabla \cdot \bar{D} = 0$

$\nabla \cdot \bar{E} = 0$.

④ $\nabla \cdot \bar{H} = 0$

$\nabla \cdot \bar{B} = 0$.

→ Taking curl on - ① both sides.

$$\nabla \times \nabla \times \bar{E} = -\mu \nabla \times \frac{\partial \bar{H}}{\partial t}.$$

$$\therefore \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\mu \, \partial/\partial t \, (\nabla \times \bar{H}).$$

$$\therefore 0 - \nabla^2 \bar{E} = -\mu \sigma \frac{\partial \bar{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}.$$

$$\therefore \boxed{\begin{array}{l} \nabla^2 \bar{E} - \mu \sigma \dfrac{\partial \bar{E}}{\partial t^2} - \mu \epsilon \dfrac{\partial^2 \bar{E}}{\partial t^2} = 0 \\ \nabla^2 \bar{H} - \mu \sigma \dfrac{\partial \bar{H}}{\partial t} - \mu \epsilon \dfrac{\partial^2 \bar{H}}{\partial t^2} = 0 \end{array}}$$

||ly

Vector Wave eqns.

→ w.r.t. the above in phasor form:

$$\nabla^2 \bar{E_s} - j\omega\mu\sigma \bar{E_s} - (j\omega)^2 \mu\epsilon \bar{E_s} = 0.$$

$\therefore$ $\boxed{\nabla^2 \bar{E_s} - \gamma^2 \bar{E_s} = 0.}$

similiarly, $\boxed{\nabla^2 \bar{H_s} - \gamma^2 \bar{H_s} = 0.}$

Wave eqn in phasor form.

where, $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$\therefore$ $\gamma = \alpha + j\beta$.

γ : propogation constant.

α : Attenuation constant (Np/m). when base e the neper will come. (in unit)

β : phase shift constant (rad/m).

* $(\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\epsilon)$.

$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon$ (real parts) —— (A)

$2\alpha\beta = \omega\mu\sigma$ (Im. parts)

$(\alpha^2+\beta^2)^2 = (\alpha^2-\beta^2)^2 + (2\alpha\beta)^2$.

$\therefore$ $\alpha^2 + \beta^2 = \sqrt{\omega^4\mu^2\epsilon^2 + \omega^2\mu^2\sigma^2}$ — (B)

Using (A) and (B) find 'α' and 'β'.

Add A+B. A+B.

$$\therefore \quad 2\alpha^2 = \omega^2 \mu^2 \sqrt{\omega^2\epsilon^2 + \omega \frac{\sigma^2}{\omega^2}} - \omega^2 \mu \epsilon .$$

$$\therefore \quad \cancel{2\alpha^2 = \omega^2\mu\epsilon\left[\sqrt{\omega^2\epsilon^2 + \sigma^2} - \mu\epsilon\right]}.$$

$$\therefore \quad \cancel{\alpha^2 = \omega \sqrt{\frac{\mu\epsilon}{2}}}$$

$$\therefore \quad \cancel{2\alpha^2 = \omega^2\mu\epsilon\left\{ \omega^2 + \frac{\sigma^2}{\epsilon^2}\right.}$$

$$\boxed{\begin{aligned} \alpha &= \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} - 1\right]} \\ \therefore \quad \beta &= \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} + 1\right]} \end{aligned}}$$

* Expanding wave eqn's in cartesian coordinates.

→ For the simplicity let us assume that the wave is propogating along z-direction in the unbounded medium. since there are ~~Since~~ no boundaries to meet along x & y directions.

We can conclude that partial variations of any field component with respect to x & y vanishing.

→ From $\nabla \cdot \bar{E} = 0$. and $\nabla \cdot \bar{H} = 0$. we can also show that E_z and H_z are zero. The vector wave eqn reduced to.

$$\frac{\partial^2 \bar{E}_s}{\partial x^2} + \frac{\partial^2 \bar{E}_s}{\partial y^2} + \frac{\partial^2 \bar{E}_s}{\partial z^2} - \gamma^2 \bar{E}_s = 0$$

$$\frac{\partial^2 \bar{H}_s}{\partial x^2} + \frac{\partial^2 \bar{E}_s}{\partial y^2} + \frac{\partial^2 \bar{E}_s}{\partial z^2} - \gamma^2 \bar{E}_s = 0.$$

These are 2nd order 3-dim PDE.

$$\therefore \xrightarrow{\partial/\partial x () = 0,\ \partial/\partial y () = 0} (2)$$

$$\therefore \cancel{\frac{\partial^2 \bar{E}}{d}} \quad \frac{d^2 \bar{E}}{dz^2} - \gamma^2 \bar{E} = 0.$$

$$\frac{d^2 \bar{H}}{dz^2} - \gamma^2 \bar{H} = 0$$

In general,

$$\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + \underbrace{E_z}_{=0} \hat{a}_z.$$

$$\bar{H} = H_x \hat{a}_x + H_y \hat{a}_y + \underbrace{H_z}_{=0} \hat{a}_z.$$

$$\rightarrow \left.\begin{aligned} &\frac{d^2 E_{xs}}{dx^2} - \gamma^2 E_{xs} = 0. \\ &\frac{d^2 E_{ys}}{dy^2} - \gamma^2 E_{ys} = 0. \\ &\frac{d^2 H_{xs}}{dx^2} - \gamma^2 H_{xs} = 0. \\ &\frac{d^2 H_{ys}}{dy^2} - \gamma^2 H_{ys} = 0 \end{aligned}\right\} \text{2nd order simple DE's}$$

→ The above eqn's is in the form of harmonic eqn. Soln of a harmonic eqn may take either sine or cosine or exponent.

Let, us consider one of the eqs.

$$\therefore \frac{d^2 E_{ys}}{dz^2} - \gamma^2 E_{ys} = 0.$$

$$\therefore m^2 - \gamma^2 = 0$$

$$m = \pm \gamma.$$

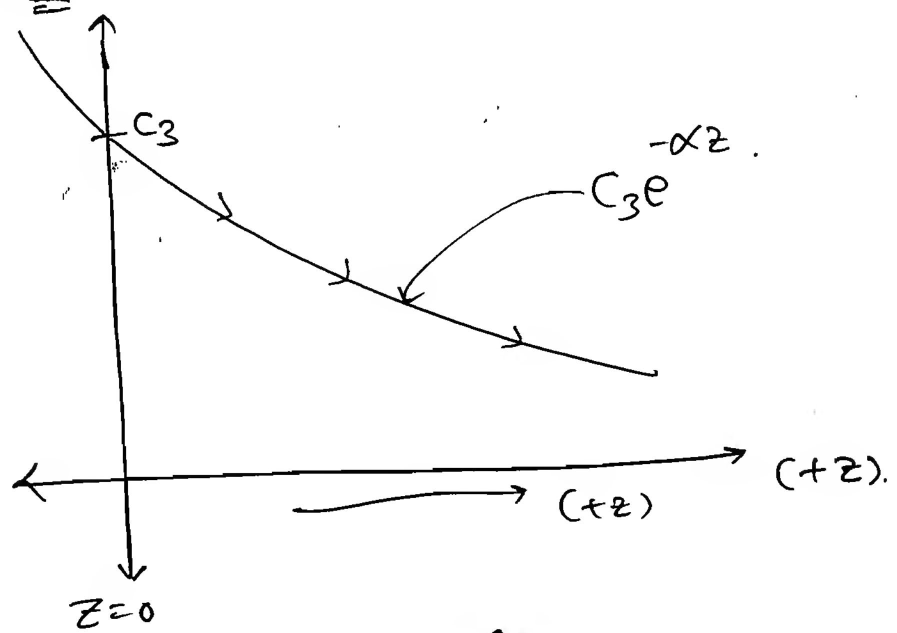
$$\therefore E_{ys} = C_3 e^{-\gamma z} + C_4 e^{+\gamma z}.$$

$$\therefore E_y(z,t) = Re\left[E_{ys} e^{j\omega t} \right].$$

$$= Re\left[C_3 e^{-\gamma z} \cdot e^{j\omega t} + C_4 \cdot e^{+\gamma z} \cdot e^{j\omega t} \right].$$

$$\therefore E_y(z,t) = Re\left[\underbrace{C_3 e^{-\alpha z} \cdot e^{j(\omega t - \beta z)}}_{\text{term-①}} + \underbrace{C_4 \cdot e^{\alpha z} \cdot e^{j(\omega t + \beta z)}}_{\text{term-②}} \right].$$

→ Term ①

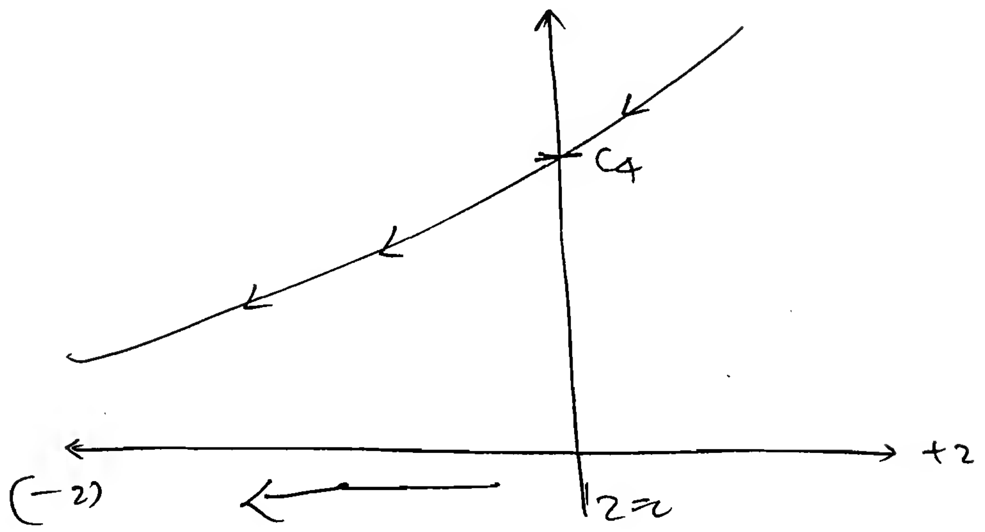


$$\rightarrow \boxed{C_4 e^{-\alpha z}} \; e^{j\boxed{(\omega t - \beta z)}}$$

Term - ②

$$C_4 e^{\alpha z} \cdot e^{j(\omega t + \beta z)}$$

$(-z) \leftarrow$



→ Term - ① In its mathamitical form indicates that wave is propogating along +z direction. While it is progressing its ampiitude is decreasing exponetially.

→ Term - ② in its mathamatical form indicates that the wave is propogating along -ve z - direction. while it is propogating its ampiitude is decrecaising.

→ For the waves which are progressing in along +ve z direction we consider term ① only and ignoring term ②

→ For the waves which are progressing along -ve z-direction we consider term-② only.

→ If the medium is perfect conductor,
in perfect conductor $\sigma = \infty$ therefore, $\alpha = \infty$.
and hence we conclude that the wave propagation through a perfect conductor is impossible. We can also conclude that ~~wave propogation~~ conducting medium is a lossy medium.

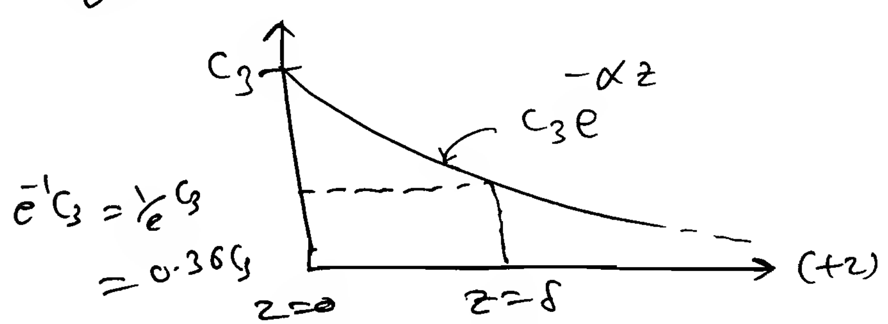
* Skin Depth 'δ'

→ It is also called depth of penetration.

→ It is that depth where the signal amplitude becomes 36 % of its original value (or) $1/e$ times of its original value.

→ Let us assume that wave is propogating along +ve z-direction and further we assume that amplitude variation is given by $C_3 e^{-\alpha z}$.

→ Further we assume that medium start from $z=0$.



$C_3 e^{-\alpha z} = C_3 e^{-\alpha \delta} = C_3 e^{-1} = 37\%$.

$\therefore$ (or) $\alpha\delta = 1$.

$$\therefore \boxed{\delta = 1/\alpha \ m}$$

Skin depth is the reciprocal of attenuation constant.

→ Skin depth in a perfect conductor is zero. because wave can not penetrate into the perfect conductor.

→ Skin depth in a lossless medium is ∞.

* <u>Impedence of the Medium:</u>

→ It is the ratio of E to H.

→ It will Purely depends upon medium properties.

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| \angle \theta_n \ \Omega.}$$

$|\eta|$ is the magnitude of η θ_n is the phase angle.

$\bar{J}_c = \sigma \bar{E}$ $\quad\quad$ $\bar{J}_D = \epsilon \frac{\partial \bar{E}}{\partial t}$

$\bar{J}_{cs} = \sigma \bar{E}_s$ $\quad\quad$ $\bar{J}_{Ds} = j\omega\epsilon \bar{E}_s$

$$\frac{\bar{J}_{cs}}{\bar{J}_{Ds}} = \frac{\sigma}{j\omega\epsilon} = \frac{\sigma}{\omega\epsilon} \angle -90^\circ$$

→ $\bar{J_c}$ is lager $\bar{J_{ds}}$ 90°.

→ $\frac{\sigma}{\omega\epsilon}$ is a bench mark to decide weither the given medium is a conductor (or) dielectric as follows:

$$\text{If } \boxed{\begin{array}{lll} \frac{\sigma}{\omega\epsilon} >> 1 & \Rightarrow & \text{Good Conductor} \\ = \infty & \Rightarrow & \text{perfect conductor} \\ = 0 & \Rightarrow & \text{perfect dielectric} \\ << 1 & \Rightarrow & \text{Good dielectric.} \end{array}}$$

* In good Conductor $\frac{\sigma}{\epsilon\omega} >> 1$.

$$\eta = \sqrt{\frac{j\omega\mu}{\left(\sigma\left(1 + j\frac{\omega\epsilon}{\sigma}\right)\right)}} \simeq \sqrt{\frac{j\omega\mu}{\sigma}} = \boxed{\sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ}$$

$$\sqrt{jr} = \sqrt{r \angle 90^\circ}$$
$$= \sqrt{r e^{j\pi/2}}$$
$$= \sqrt{r} \cdot \sqrt{e^{j\pi/4}}$$
$$= \sqrt{r} \cdot \angle 45^\circ$$

(i.e) In a good conductor E leads H by 45°.

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}} \text{ NP/m.}$$

similarly

$$\boxed{\beta = \sqrt{\frac{\omega\mu\sigma}{2}}} \text{ rad/m.}$$

⇒ Skin depth

$$\therefore \boxed{\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \text{ m.}}$$

$\frac{\sigma}{\omega\epsilon}$ is also ~~di~~ called dissipication factor (or) loss tangent.

$$\phi = \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right).$$

$$\boxed{\tan\phi = \frac{\sigma}{\omega\epsilon} = \beta/\alpha}.$$

It can be proved that $\boxed{\phi = 2\theta_n}$

Ex-1 In a lossy medium the Impedence of the medium is 200 at an angle of 30° and $\bar{H} = 10e^{-\alpha x}\cos(\omega t - \frac{x}{2})\hat{a}_z$ A/m.

Find attenmation constant α, β, & direction of propogation. Expression for $\bar{E}$.

Ans:

$\eta = 200 \angle 30°$

$$\xrightarrow{(\omega t - x)} (+x)$$

$\boxed{\beta = +\frac{1}{2}}$ rad/m

$$\boxed{\frac{E_y}{H_z} = \eta} = \frac{-E_z}{H_y}$$

$\therefore$ $E_y = \eta H_z.$

$\therefore$ $E_y = 2000 \angle 30° \cdot e^{-\alpha x} \cdot \cos(\omega t - \frac{x}{2})\hat{a}_z.$

$\therefore$ $E_y = 2000 e^{-\alpha x} \cdot \cos \cos[\omega t - \frac{x}{2} + \frac{\pi}{6}]\hat{a}_z.$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}} + 1\right]}.$$

$$\Rightarrow \quad \frac{\alpha}{\beta} =$$

$$\phi = 2\theta_n = 2(30^\circ) = 60^\circ$$

$$\therefore \quad \tan\phi = \frac{\sigma}{\omega\epsilon}$$

$$\therefore \quad \frac{\sigma}{\omega\epsilon} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \alpha = \beta/\sqrt{3}$$

$$\therefore \quad \boxed{\alpha = \frac{1}{2\sqrt{3}}} \quad Np/m$$

$$\therefore \quad \delta = \frac{1}{\alpha}$$

$$\therefore \quad \boxed{\delta = 2\sqrt{3}\ m}$$

Ex-2 In a good Conductor attenuation Constant α is given by 0.5 np/m. Find phase shift constant and skin depth.

Ans: $\beta = 0.5$ $\quad \alpha = \beta$ for good Conductor.

$\beta = 0.5$ rad/m

$\therefore \delta = 2$ m.

If α is in the dB/m we have to Converted in np/m.

☆ Polarization:

→ It is the one of the designed parameter of an antenna. and it is the of a wave.

→ It is defined as orientation of electric field (or) It is the locus of resultant electric field as a function of time at a given location.

→ If the Locus represents a straight line then it is called linear polarization.

→ VP and HP are the special cases in the linear polarization.

→ If the Locus represent a circle then it is called circular polarization.

→ If the Locus represent an ellipse then it is called elliptical polarization.

→ Right hand and Left hand are two different kinds in CP and in E.P.

→ For example we assumed that the wave is propagating along z direction. The possible non-zero transverse fields components are $\hat{a}_x$ and $\hat{a}_y$

$\longrightarrow (+z)$

$$\bar{E}(z,t) = E_{xo}\cos(\omega t - \beta z)\,\hat{a}_x + E_{yo}\cos(\omega t - \beta z + \phi_p)\,\hat{a}_y.$$

→ phase shift b/w the transverse field component is ϕ_p.

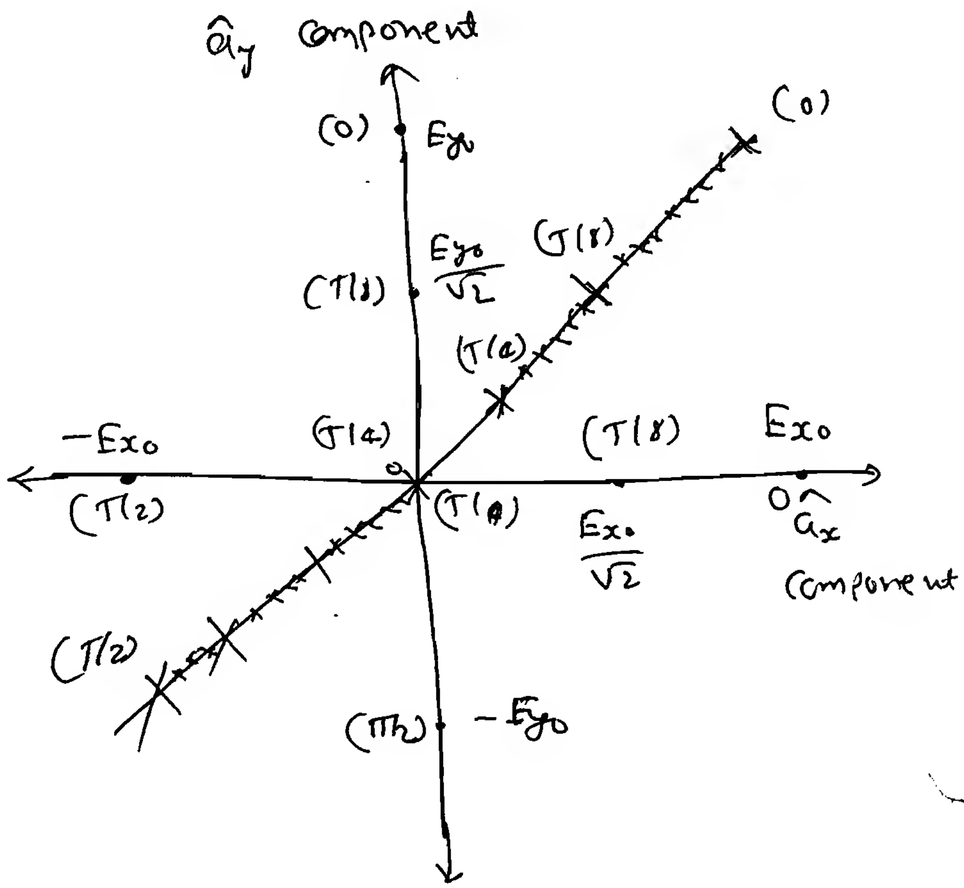
→ For convience choose $z=0$.

$$\therefore \bar{E}(0,t) = E_{xo}\cos\omega t\,\hat{a}_x + E_{yo}\cos(\omega t + \phi_p)\,\hat{a}_y.$$

$\therefore$ $\boxed{\underline{\text{Case - 1}}}$ $\phi_p = 0$.

$$\therefore \bar{E} = E_{xo}\cos\omega t\,\hat{a}_x + E_{yo}\cos\omega t\,\hat{a}_y.$$

t	$\omega t = \frac{2\pi}{T}t$
0	0
$\vdots$	
$\frac{T}{8}$	$\pi/4$
$\vdots$	
$\frac{T}{4}$	$\pi/2$
$\frac{T}{2}$	π
$\vdots$	$\vdots$
T	2π



→ Thus, the Locus represents a straight line and hence the wave is said to be linearly polarized.

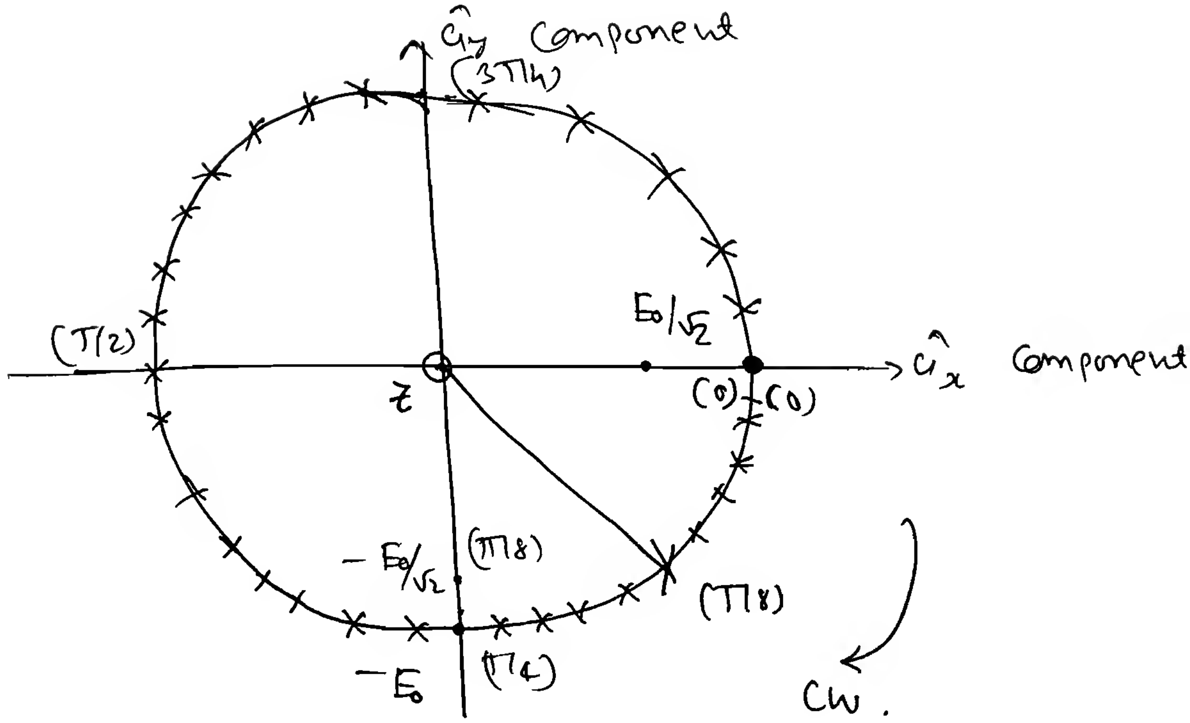
→ If a_y component is absent the ~~the~~ locus lies along ~~vertical~~ Horizontal axis that we may call wave is horizontally Polarized.

→ If a_x component is absence the locus lies along vertical axis that we may call wave is ~~f~~ Vertically polarized.

→ ✓ ☆ To achieve linear polarization the wave must have atleast one transfers field components. If the wave has both the transfers field components then the phase difference betⁿ them must be equal to $\pm n\pi$ when $n = 0, 1, 2, \ldots$

$\boxed{\text{Case- 2:}}$ $\phi_p = 90^\circ$, $E_{x0} = E_{y0} = E_0$

$$\bar{E} = E_0 \cos\omega t \, \hat{a}_x - E_0 \sin\omega t \, \hat{a}_y.$$



CCW → RHCP.

CW → LHCP.

→ Locus represents a ~~circular~~ circle and as shown in the figure it is rotating in the clock wise direction. while the wave propagates it the locus rotates in the CCW Direction then it is called RHCP.

→ If the locus rotates in the clock wise direction the it is called LHCP.

→ For resulting circular Polarization the wave must have two transfers fields components. They must have equal amplitudes and the phase difference betn them musts be equals to $\boxed{\pm \frac{n\pi}{2}}$ where 'n' is odd.

→ To achive elliptical polarization the wave must have two transvers fields component, they must have unequal amplitudes and the phase difference betⁿ them must not equal to 0° or 180°.

Ex-1 Identify the polarization of the following.

1. $\bar{E} = 20\cos(\omega t - \frac{x}{2})\,\hat{a}_y$ V/m.

2. $\bar{E} = 45\sin(\omega t - \beta x)\,\hat{a}_y + 45\sin(\omega t - \beta x)\,\hat{a}_z$.

3. $\bar{E} = 25\sin(\omega t - \beta z)\,\hat{a}_x + 25\cos(\omega t - \beta z)\,\hat{a}_y$.

4. $\bar{E} = 35\sin(\omega t - \beta y)\,\hat{a}_x + 45\cos(\omega t - \beta y)\,\hat{a}_z$.

5. $\bar{E} = Re\{[2\hat{a}_x + 3j\hat{a}_y]\,e^{j(\omega t - \beta z)}\}$.

6. $\bar{E} = 25\cos(\omega t - \beta r)\,\hat{a}_\theta$ V/m.

7. $\bar{H} = 20\sin(\omega t - \beta x)\,\hat{a}_z$

8. $\bar{E} = 20\sin(\omega t - \beta x)\,\hat{a}_y + 20\sin(\omega t - \beta x + 90°)\,\hat{a}_z$.

9. $\bar{E} = 20\sin(\omega t - \beta z)\,\hat{a}_z$.

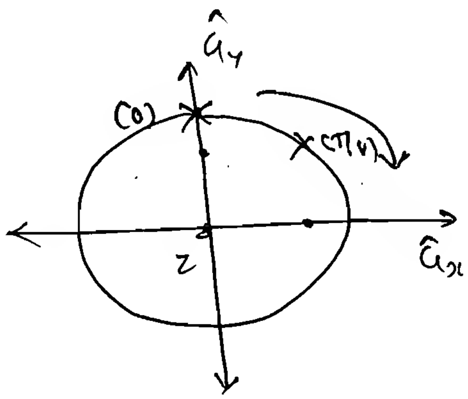
10. $\bar{E} = Re\{[\hat{a}_y + j\hat{a}_z]\}\,e^{j(\omega t - \beta x)}\}$.

Ans:-1

→ The wave is propagating along +x direction and it has one transverse fields component then for it is linearly polarized. and it is polarized along y direction. because the electric field has $\hat{a}_y$ component.

Ans-2 I wave is propagating along +x direction. and phase difference is zero. so it is linearly polarized.

Ans:-3 circularly polarization. ---→ +(z).

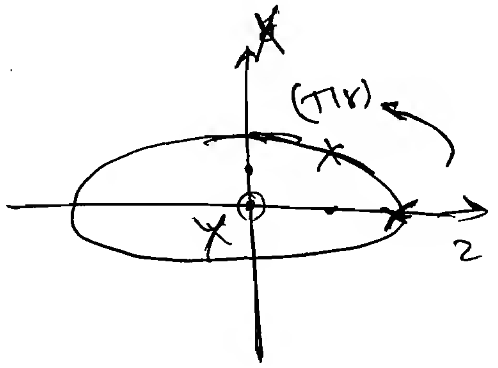


LHCP

Ans:- 4 $\bar{E} = 35 \sin(\omega t - \beta y)\hat{a}_x + 45\cos(\omega t - \beta y)\hat{a}_z$.

$35 \neq 45$

---→ (+y) so, Eliptical.



RHEP

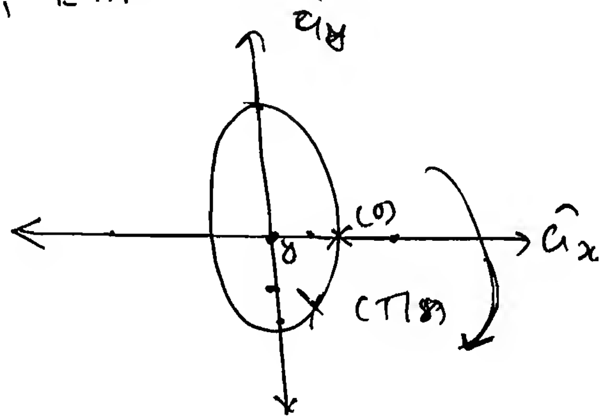
5. $E = Re\{(2\hat{a}_x + 3j\hat{a}_y).[\cos(\omega t - \beta z) + j\sin(\omega t - \beta z)]\}$

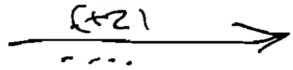
$\therefore E = 2\cos(\omega t - \beta z)\hat{a}_x - 3\sin(\omega t - \beta z)$.

$= \cancel{2\cos(\omega t - \beta z)\hat{a}_x + 3\cos(\omega t - \beta z + \frac{\pi}{2})\hat{a}_y}$

$2 \neq -3$

So, Elliptical.





LHEP.

6. $\bar{E} = 25\cos(\omega t - \beta r)\hat{a}_\theta$ V/m.

where, r, θ spherical coordinates.

→ Linearly polarized. along θ direction.

→ This represents a wave which is propagating along radial direction i.e. $\hat{a}_r$ direction. Therefore, the transfers plane would be r = constant plane (or) θ - ϕ plane.

→ Therefore, the transfers field components are $\hat{a}_\theta$ and $\hat{a}_\phi$. given electric field has one transfer fields components i.e. $\hat{a}_\theta$ component and hence wave is linearly polarized along $\hat{a}_\theta$ direction.

polarized and is

7. $\bar{H} = 20 \sin(\omega t - \beta x)\,\hat{a}_z$.

$\longrightarrow +x$

→ Linearly polarized.

$\bar{E} = 20\eta \sin(\omega t - \beta x)\,\hat{a}_y$ → y-direction.

→ This wave is linearly polarized and is polarized along y-direction because electric field have y-component.

8. $\bar{E} = 20 \sin(\omega t - \beta x)\,\hat{a}_y + 20 \sin(\omega t - \beta x + 40)\,\hat{a}_z$.

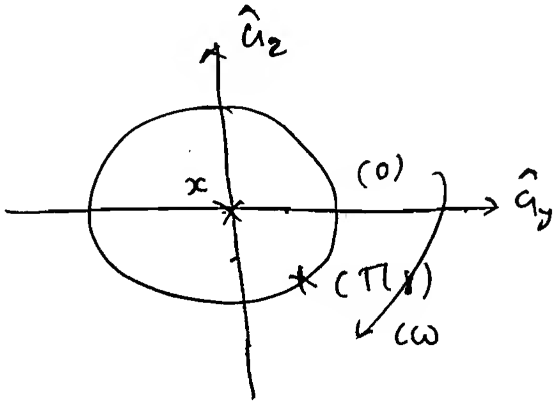
Ans: This wave may be valid em wave but it is not satisfying any polarization principle.

9. $\bar{E} = 20 \sin(\omega t - \beta z)\,\hat{a}_z$

→ Not valid EM wave

10. $\bar{E} = \mathrm{Re}\left\{ [\hat{a}_y + j\hat{a}_z] \cdot \cos(\omega t - \beta x) + j(\sin(\omega t - \beta x)) \right]$.

$\bar{E} = \cos(\omega t - \beta x)\,\hat{a}_y - (\sin(\omega t - \beta x))\,\hat{a}_z$.



LHCP

★ POYNTING THEOREM / VECTOR.

→ This used to calculate average power.

→ The vector corresponds to this indicates direction of instaneous energy flow (or) simply direction of propogation.

→ If $\bar{E}$, $\bar{H}$ are electrical and magnetic field respectively then the instanneous poynting vector is given by cross $\bar{E} \times \bar{H}$.

$$\boxed{\bar{P}_{instanteneous} = \bar{E} \times \bar{H}.}$$

* The avg. poynting vector is given by.

$$\boxed{\bar{P}_{avg} = \frac{1}{2} \bar{E}_s \times \bar{H}_s^* \quad W/m^2}$$

$*$ → Complex conjugate

→ $\bar{E}_s$, $\bar{H}_s$ are the fields in the phasor form. The avg. power crossing a cross sectional surface of 'S' is given by,

$$\therefore \boxed{W_{avg} = \int_s \bar{P}_{avg} \cdot d\bar{s} \quad \text{watts.}}$$

$d\bar{s}$: vector differential surface element and will be projecting normal to the surface.

Ex-1 In a non-magnetic medium $\bar{E} = 8\cos(2\pi \times 10^7 t - 0.8x)\,\hat{a}_y$ V/m. Find the relative permitivity of the medium, intrinsic impedence of the medium, $\bar{H}$, Polarization and also calculate the amount of power crossing a 100 cm² area. difine on the plane x=1.

Ans:

$$\bar{E} = 8\cos(2\pi \times 10^7 t - 0.8x)\,\hat{a}_y$$

$\rightarrow$ $\omega = 2\pi \times 10^7$ rad/s.

$\beta = 0.8$

non-magnetic medium

$\therefore \mu = \mu_0$

$\epsilon = \epsilon_0 \epsilon_r$.

$\longrightarrow (+x)$

$\therefore \beta = \omega\sqrt{\mu\epsilon}$.

$\therefore \beta = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r}$

$$\boxed{\frac{E_y}{H_z} = \eta} = -\frac{E_z}{H_y}$$

$\therefore \sqrt{\epsilon_r} = \dfrac{\beta}{\omega\sqrt{\mu_0\epsilon_0}}$.

$\therefore \sqrt{\epsilon_r} = \dfrac{0.8 \times 3\times10^8}{2\pi \times 10^7}$.

$\therefore \boxed{\epsilon_r = 14.60}$

$\therefore \eta = \sqrt{\dfrac{\mu}{\epsilon}}$

$\therefore \eta = \sqrt{\dfrac{\mu_0}{\epsilon_0 \epsilon_r}}$

$\therefore \eta = \dfrac{120\pi}{\sqrt{\epsilon_r}}$

$\therefore \boxed{\eta = 98.61}$

$\therefore H_z = \dfrac{E_y}{\eta}$.

$$\therefore \boxed{\bar{H} = \frac{8}{98.61}\cos(2\pi\times10^7 t - 0.8x)\,\hat{a}_z}$$

$\bar{H} = 0.08113\cos(2\pi\times10^7 t - 0.8x)\,\hat{a}_z$.

$\rightarrow \quad \bar{P}_{avg} = \frac{1}{2}\bar{E}_s \times \bar{H}_s$.

$$\therefore \quad \bar{E}_s = 8 \cdot e^{-j0.8x}\,\hat{a}_y .$$

$$\bar{H}_s = \frac{8}{\eta}\cdot e^{+j0.8x}\cdot \hat{a}_z .$$

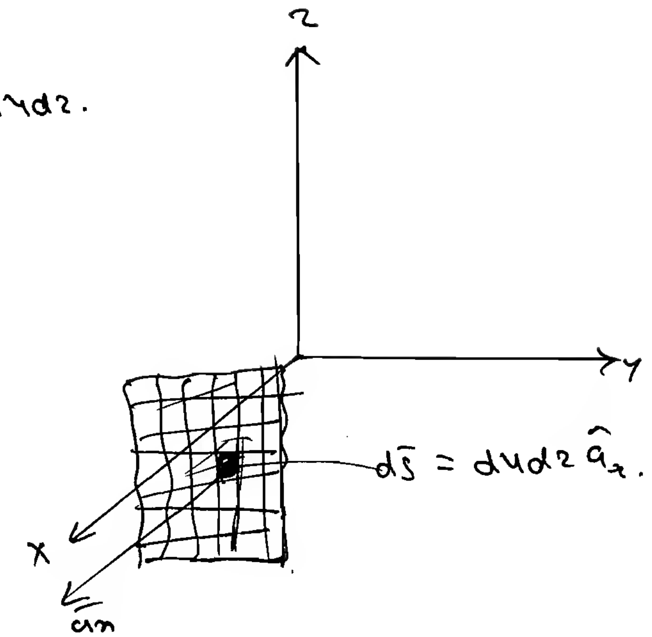
$$\therefore \quad \bar{P}_{avg} = \frac{1}{2}\left(\bar{E}_s \times \bar{H}_s\right)$$

$$= \frac{1}{\cancel{2}} \times \frac{\overset{32}{\cancel{64}}}{\eta} \times \hat{a}_x$$

$$\therefore \quad \bar{P}_{avg} = \frac{32}{98.6} \times \hat{u}_x$$

$$\therefore \quad \boxed{\bar{P}_{avg} = 0.3245\,\hat{a}_x}$$

$$\therefore \quad \bar{W}_{avg} = \int \bar{P}_{avg}\cdot d\bar{S}$$



$$\therefore \quad \bar{P}_{avg}\cdot d\bar{S} = 0.3245\; dy\,dz.$$

$$\therefore \quad \bar{W}_{avg} = \int_S 0.3245\; dy\,dz.$$

$$= 0.3245 \int_S dy\,dz$$

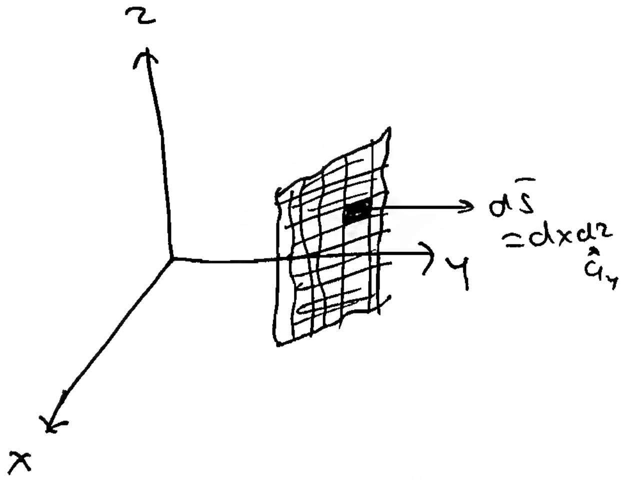
$$\therefore \quad \boxed{\bar{W}_{avg} = 3.245\times 10^{-3}\ \text{Watts}}$$

$$\int_S dy\,dz = 100\,cm^2$$

$$= \frac{100}{100\times 100}\,m^2.$$

Ex-2 Repeat the above example to calculate avg. power crossing 100 cm^2 area define on the plane $y = 1m$.

Ans: zero



$$d\bar{s} = dx\,dz\,\hat{a}_y$$

$$\therefore \bar{P}_{avg} \cdot d\bar{s} = 0.$$

$$\therefore \boxed{\bar{W}_{avg} = 0.}$$

Ex-2 Repeat the above problem to calculate the amount of power crossing through a 50 cm^2 area define on the plane $2x + 3y = 5$.

Ans: $2x + 3y = 5.$

$$\therefore \hat{a}_n = \frac{2\hat{a}_x + 3\hat{a}_y}{\sqrt{13}}$$

$$\therefore d\bar{s} = ds\,\hat{a}_n.$$

$$\therefore \bar{P}_{avg} \cdot d\bar{s} = 0.3245\,\hat{a}_x \cdot \left(\frac{2\hat{a}_x + 3\hat{a}_y}{\sqrt{13}}\right).$$

$$= 0.18\,ds.$$

$$\therefore W_{avg} = \int_s \bar{P}_{avg} \cdot d\bar{s}.$$

$$\therefore W_{avg} = \int 0.18\,ds$$

$$= 0.18 \int ds$$

$$W_{avg} = 0.18 \times \frac{50 \times \cancel{100}}{\cancel{100} \times 100}$$

$$\therefore \boxed{W_{avg} = 0.09 \text{ watts.}}$$

Ex. 3 Repeat the above problem to find the avg. power crossing on a circular disk of radius 2m define on the plane $x = 2m$.

Ans: $\bar{P}_{avg} = 0.3245 \, \hat{a}_x$.

$$\therefore d\bar{s} = dy\,dz \, \hat{a}_x .$$

$$W_{avg} = \int_s \bar{P}_{avg} \cdot d\bar{s}$$

$$= \int_s 0.3245 \, dy\,dz$$

$$= 0.3245 \int_s ds.$$

$$= 0.3245 \times \pi r^2$$

$$= 0.3245 \times \pi \times (2)^2.$$

$$\therefore \boxed{W_{avg} = 4.075 \text{ watts}}$$

☆ Reflection:

→ When a wave is progressing from one medium to other medium impedence of a medium Changes. Change in the impedence of the medium is said to be a impedence discontinuity (or) Impedence are irregular (or) not uniformly.

→ Wherever there exist impedence discontinuity then part of the energy will be transmitted and the part of the energy will be reflected.

→ If the vector corresponds to the direction of propagation is normal to the interface then it is called normal incidence.

→ If the vector corresponds to direction of propagation makes an angle with the unit vector normal to the interface is called oblique incidence.

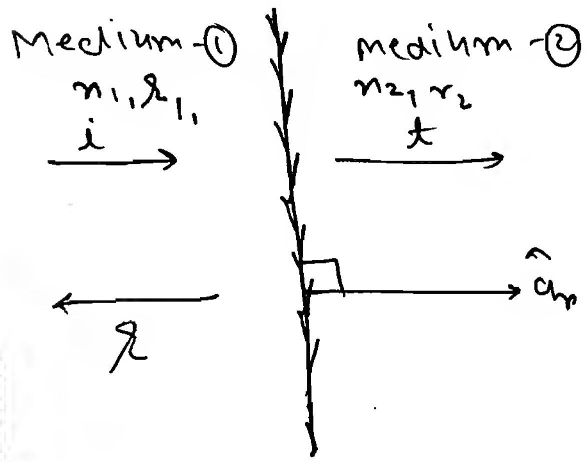
→ There are two types of oblique incidence.

① Parallel Polarization.

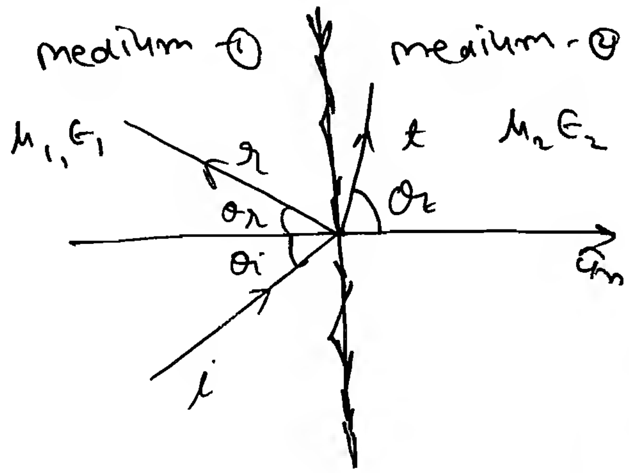
② Perpendicular Polarization.

Reflection

Normal incidence | Oblique incidence



interface.



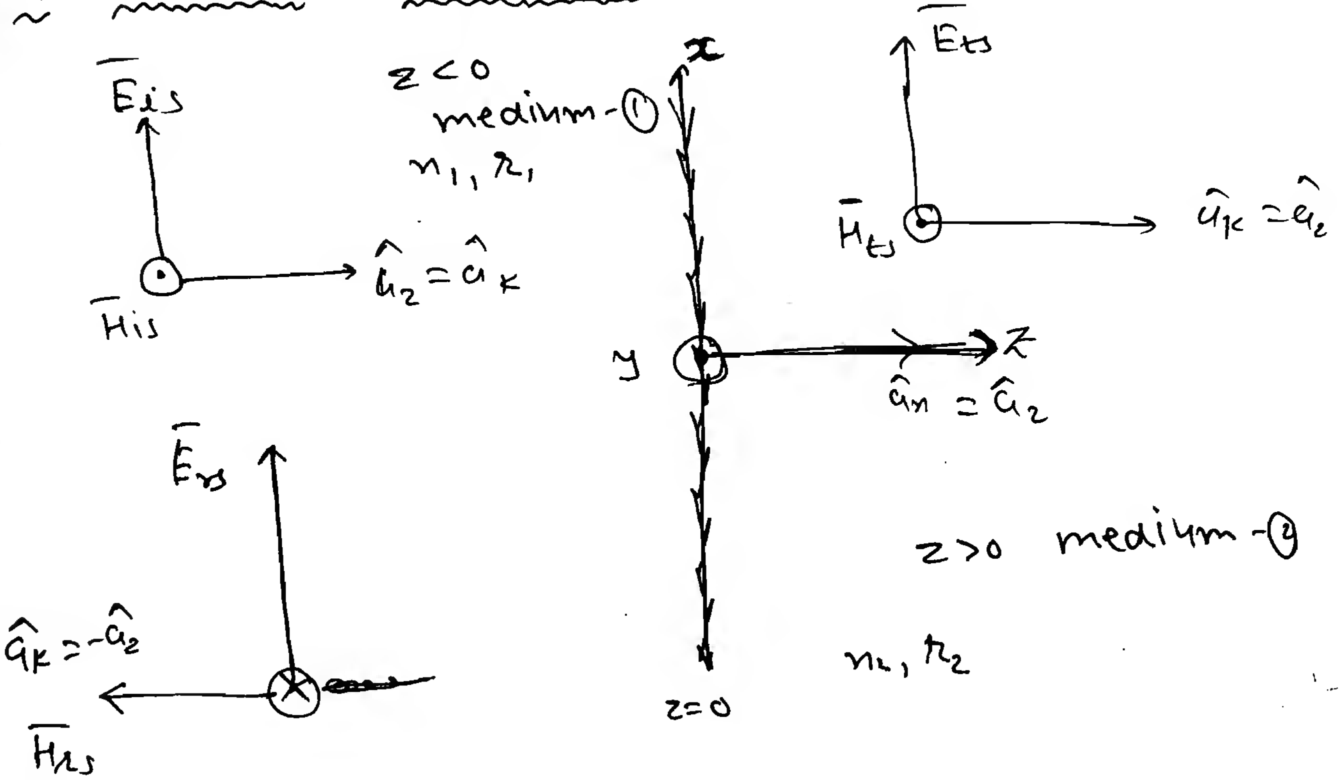
Snell's Law

$$\theta_i = \theta_r.$$

$$\therefore \boxed{\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}}$$

$\rightarrow i$

* Normal Incidence:



→ i = incident
r = reflected
t = transmitted

$\hat{a}_k$: vector corresponds to direction of propagation.

→ For the incident $\hat{a}_k = +\hat{a}_z$

→ For the transmitted $\hat{a}_k = +\hat{a}_z$

→ For the reflected $\hat{a}_k = -\hat{a}_z$

$\odot$ vector / axis coming out of the paper.

$\otimes$ vector / axis going into the paper.

→ Figure shows that an interface define by $z=0$.

→ $z<0$ is medium-1 and is characterised by η_1, γ_1.

→ Medium-2 is define for $z>0$ and is characterised by η_2, γ_2.

→ We assume that ~~wave~~ wave is progressing from medium-1 to medium-2.

→ Because of Impedence discontinuity part of the energy will be transmitted and part of the energy will be reflected.

* incident $\xrightarrow{\hspace{3cm}}$ (+z)

$$\bar{E}_{is} = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x$$

$$\bar{H}_{is} = \frac{E_{io}}{\eta_1} \cdot e^{-\gamma_1 z}\, \hat{a}_y .$$

* Transmitted $\xrightarrow{\hspace{3cm}}$ (+z)

$$\bar{E}_{ts} = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x .$$

$$\bar{H}_{ts} = H_{to}\, e^{-\gamma_2 z}\, \hat{a}_y .$$

$$\bar{H}_{ts} = \frac{E_{to}}{\eta_2} \cdot e^{-\gamma_2 z}\, \hat{a}_y .$$

* Reflected $\xrightarrow{\hspace{3cm}}$ (−z).

$$\therefore \bar{E}_{rs} = \bar{E}_{ro} \cdot e^{+\gamma_1 z}\, \hat{a}_x$$

$$\bar{H}_{rs} = -\frac{E_{ro}}{\eta_1} \cdot e^{+\gamma_1 z}\, \hat{a}_y .$$

→ In medium −① across the interface there are two sets of fields (i) incidence (ii) Reflected.

→ In medium −② there exist one set of fields i.e. transmitted.

→ The electric and magnetic fields of this waves are projecting tangential to the interface.

→ We know that tangential components of electric field are continuous across a ~~current~~ the interface.

→ Similary tangential components of magnetic field are also continous across a current free interface.

→ As shown in the figure, the interface is define by $z = 0$.

→ $E_{io} + E_{ro} = E_{to}$ — ① (continuity of E-fields).

$\therefore \dfrac{E_{io}}{\eta_1} - \dfrac{E_{ro}}{\eta_1} = \dfrac{E_{to}}{\eta_2}$ — ② (continuity of H-fields).

→ defining reflection co-efficient Γ as a ratio of reflected electric field to the incident electric field.

$$\therefore \Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}.$$

$\therefore$ $\cancel{E_{io} + E_{ro} = E_{to}}$

$\cancel{E_{io} - E_{ro} = \frac{\eta_1}{\eta_2} E_{to}}$

$$E_{io} + E_{ro} = \cancel{E_{to}}$$

$$\frac{\eta_2}{\eta_1} E_{io} - \frac{\eta_2}{\eta_1} E_{ro} = \cancel{E_{to}}$$

$$E_{io}\left[\frac{\eta_1 - \eta_2}{\eta_1}\right] + E_{ro}\left[\frac{\eta_1 + \eta_2}{\eta_1}\right] = 0$$

$$\therefore E_{io}[\eta_2 - \eta_1] = E_{ro}[\eta_2 + \eta_1].$$

$$\therefore \boxed{\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}.}$$

→ Defining transmission co-efficient of T is a is a ratio of transmitted $\bar{E}$ ~~wave~~ to the incident electric field.

$$T = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_1 + \eta_2} = 1 + \Gamma.$$

$$\rightarrow E_{io} + E_{ro} = E_{to}.$$

$$\rightarrow \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}.$$

$$\therefore \begin{array}{l} E_{io} + \cancel{E_{ro}} = E_{to} \\ + \\ E_{io} - \cancel{E_{ro}} = \frac{\eta_1}{\eta_2} E_{to} \\ \hline 2 E_{io} = \left[\frac{\eta_1 + \eta_2}{\eta_2}\right] E_{to}. \end{array}$$

$$\therefore \boxed{T = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

→ If $\underline{\eta_2 = \eta_1}$ ⇒ ~~zero~~ $\underline{\Gamma = 0, T = 1}$ ⇒ <u>No Reflection</u>.

⇒ No Impedence continuity.

→ If $\underline{\eta_2 = 0}$ ⇒ (i.e) <u>Medium ② is Perfect</u> conductor.

then $\underline{\Gamma = -1, T = 0}$

⇒ <u>Completely reflected</u>, <u>No transmission</u>.

→ If Γ is reflection co-efficient then $\boxed{|\Gamma|^2}$ is called power reflection coefficient.

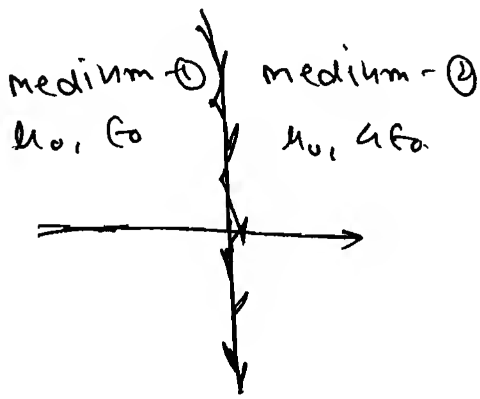
→ $\boxed{\%\text{ of power reflected} = 100\,|\Gamma|^2\,\%}$

→ $\boxed{\%\text{ of power transmitted} = \cancel{1-100}\ (1-|\Gamma|^2)\times 100\,\%}$

Ex-1 Assume normal incidence wave is progressing from free space to a medium whose permitivity is $4\epsilon_0$ and permeability is μ_0. Find the reflected power, Γ, T, power reflection coefficient, % power reflected, and also find % power transmitted.

Ans:

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$$



$$\eta_2 = \sqrt{\frac{\mu}{\epsilon}}$$

$$= \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{120\pi}{2}$$

$$\therefore \boxed{\eta_2 = 60\pi}$$

$$\therefore \Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{60\pi - 120\pi}{180\pi} = \frac{-60\pi}{180\pi} = -1/3.$$

$$\therefore \boxed{\Gamma = -1/3}$$

$$\therefore T = 1+\Gamma = 1 - 1/3 = 2/3.$$

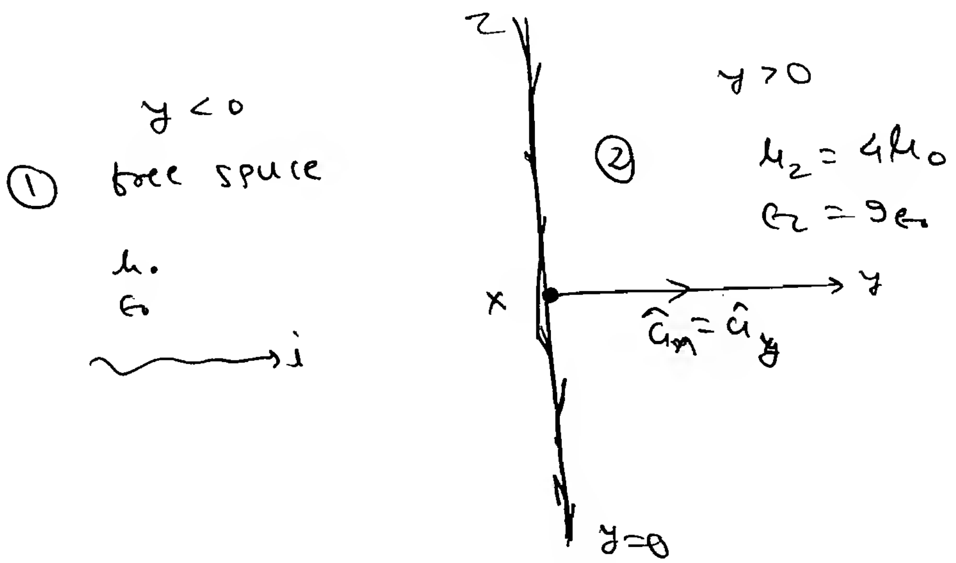
$$\boxed{T = 2/3}$$

$|\Gamma|^2 = 1/9 \Rightarrow$ power reflection coefficient.

$\rightarrow$ % power reflected $= 100 |\Gamma|^2 = \frac{100}{9} = 11.11\%$

Ex-2 Figure shows that an interface may be define by $y = 0$, $y < 0$ is medium-① and is free space and $y > 0$ is medium-② and is characterised by $\mu_2 = 4\mu_0$ & $\epsilon_2 = 9\epsilon_0$. A wave is incident upon the interface whose electric field is given by $20 \sin(\omega t - \frac{y}{2})\, \hat{a}_z$ V/m. Find the phase shift constant in medium ① and in medium ② also find Γ, T, power reflection coefficient, % power reflected, % power transmitted, $\bar{S}$, $\bar{H}_i$, $\bar{E}_i$, $\bar{H}_r$, $\bar{E}_t$, $\bar{H}_t$ and also calculate poynting vector of incident, reflected and transmitted wave.

Ans:



z-x plane.

→ $E_i = 20 \sin(\omega t - \frac{y}{2})\,\hat{a}_z$ V/m.

→ The unit vector normal to the interface is $\hat{a}_y$.

→ From the given $\overline{E_i}$ we can conclude that the wave is progressing along +y direction which is projecting normal to the interface. and hence this case is said to be normal incedence.

→ $\beta_1 = \omega\sqrt{\mu_1 \epsilon_1}$

But $\boxed{\beta_1 = 1/2.}$ rad/s

$\therefore \beta_1 = \omega\sqrt{\epsilon_0 \mu_0}.$

$\therefore \frac{1}{2} = \omega\sqrt{\mu_0 \epsilon_0}.$

→ $\beta_2 = \omega\sqrt{\mu_2 \epsilon_2}$

$\beta_2 = \omega\sqrt{9\epsilon_0 \times 4\mu_0}.$

$\therefore \beta_2 = 6\,\omega\sqrt{\epsilon_0 \times \mu_0} = 6 \times 1/2. = 3.$

$\therefore \boxed{\beta_2 = 3}$ rad/s

$\therefore \eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$

$\boxed{\eta_2 = 80\pi}$ Ω

$\boxed{\eta_1 = 120\pi}$ Ω

$\eta_2 = \sqrt{\mu/\epsilon}.$

$\therefore \eta_2 = \sqrt{\frac{4\mu_0}{9\epsilon_0}} = 120\pi \times 2/3 = 80\pi$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\therefore \Gamma = \frac{80\pi - 120\pi}{80\pi + 120\pi} = \frac{-40\pi}{200} = -1/5.$$

$$\therefore \boxed{\Gamma = -1/5.}$$

$$\therefore \tau = 1 + \Gamma$$

$$= 1 - \frac{1}{5}.$$

$$\therefore \boxed{\tau = \frac{4}{5}}$$

$$\therefore \bar{E}_i = 20 \sin(\omega t - y/2)\hat{a}_z$$

$\therefore$ Now.

$$\longrightarrow (+y)$$

$$\therefore \boxed{\frac{E_z}{H_x} = +\eta_1} = -\frac{E_x}{H_z}.$$

$$\therefore \bar{H}_i = +\frac{\bar{E}_i}{\eta_1}$$

$$\therefore \boxed{\bar{H}_i = +\frac{20}{120\pi} \sin(\omega t - y/2)\hat{a}_x}$$

$$\rightarrow \Gamma = \frac{E_r}{E_i}.$$

$$\therefore \bar{E}_r = \Gamma \bar{E}_i$$

$$\therefore \bar{E}_r = -\frac{20}{5} \sin(\omega t + \beta_1 y)\hat{a}_z.$$

$$\therefore \boxed{\bar{E}_r = -4 \sin(\omega t + \beta_1 y)\hat{a}_z}$$

$$\xrightarrow{\hspace{4cm}} (-y).$$

$$\frac{E_z}{H_x} = -\eta_1 = -\frac{E_x}{H_z}.$$

$$\therefore \quad \bar{H}_r = -\frac{\bar{E}_i}{\eta_1}.$$

$$\therefore \quad \bar{H}_r = -\frac{1}{\frac{120\pi}{3}} \times (-4 \sin(\omega t + \beta y)\,\hat{a}_x)$$

$$\therefore \quad \boxed{\bar{H}_r = \frac{1}{30\pi} \sin(\omega t + \beta y)\,\hat{a}_x}$$

$$\therefore \quad T = \frac{\bar{E}_t}{\bar{E}_i}$$

$$\therefore \quad \bar{E}_t = T\,\bar{E}_i$$

$$\therefore \quad \bar{E}_t = \frac{4}{5} \times 4 \sin(\omega t \mp \beta_2 y)\,\hat{a}_z.$$

$$\therefore \quad \bar{E}_t = \frac{16}{5} \sin(\omega t \mp \beta_2 y)\,\hat{a}_z.$$

$$\therefore \quad \xrightarrow{\hspace{4cm}} (+y)$$

$$\frac{E_z}{H_x} = -\eta_2 = \frac{E_x}{H_z}.$$

$$\therefore \quad \bar{H}_t = -\frac{\bar{E}_t}{\eta_2} = \frac{-16}{5 \times \frac{80\pi}{5}} \sin(\omega t + \beta_2 y)\,\hat{a}_z.$$

$$\therefore \quad \boxed{\bar{H}_t = \frac{-1}{25\pi} \sin(\omega t \pm \beta_2 y)\,\hat{a}_z}$$

$\rightarrow |\Gamma|^2 = \frac{1}{25}.$

$\therefore$ % of power reflected $= 100 \times 1/25\ \% = 4\ \%.$

$\therefore$ % of Power transmitted = 96%.

$\therefore$ ① $\overline{E_{is}} = 20\, e^{-j\beta_1 y}\, \hat{a}_z$

$\overline{H_{is}^{*}} = -\frac{1}{6\pi}\, e^{+j\beta_1 y}\, \hat{a}_x.$

$\therefore \overline{P_{avg}} = \frac{1}{2}\, \overline{E_{is}} \times \overline{H_{is}^{*}}.$

$= \frac{1}{2} \times \cancel{20}^{10} \times \frac{-1}{6\pi} \times \hat{a}_y$

$\therefore \boxed{\overline{P_{avg}} = -\frac{5}{3\pi}\, \hat{a}_y}$ ω/m^2

② $\overline{E_{rs}} = -4 e^{j\beta_1 y}\, \hat{a}_z$

$\overline{H_{rs}^{*}} = \frac{1}{30\pi}\, e^{j\beta_1 y}\, \hat{a}_x.$

$\therefore \overline{P_{avg}} = \frac{1}{2}\, \overline{E_{rs}} \times \overline{H_{rs}^{*}}.$

$\therefore \overline{P_{avg}} = \frac{1}{\cancel{2}} \times -\cancel{4}^{2} \times \frac{1}{\cancel{30}\pi_{15}} \times \hat{a}_y$

$\therefore \boxed{\overline{P_{avg}} = \frac{-1}{15\pi}\, \hat{a}_y}$ ω/m^2

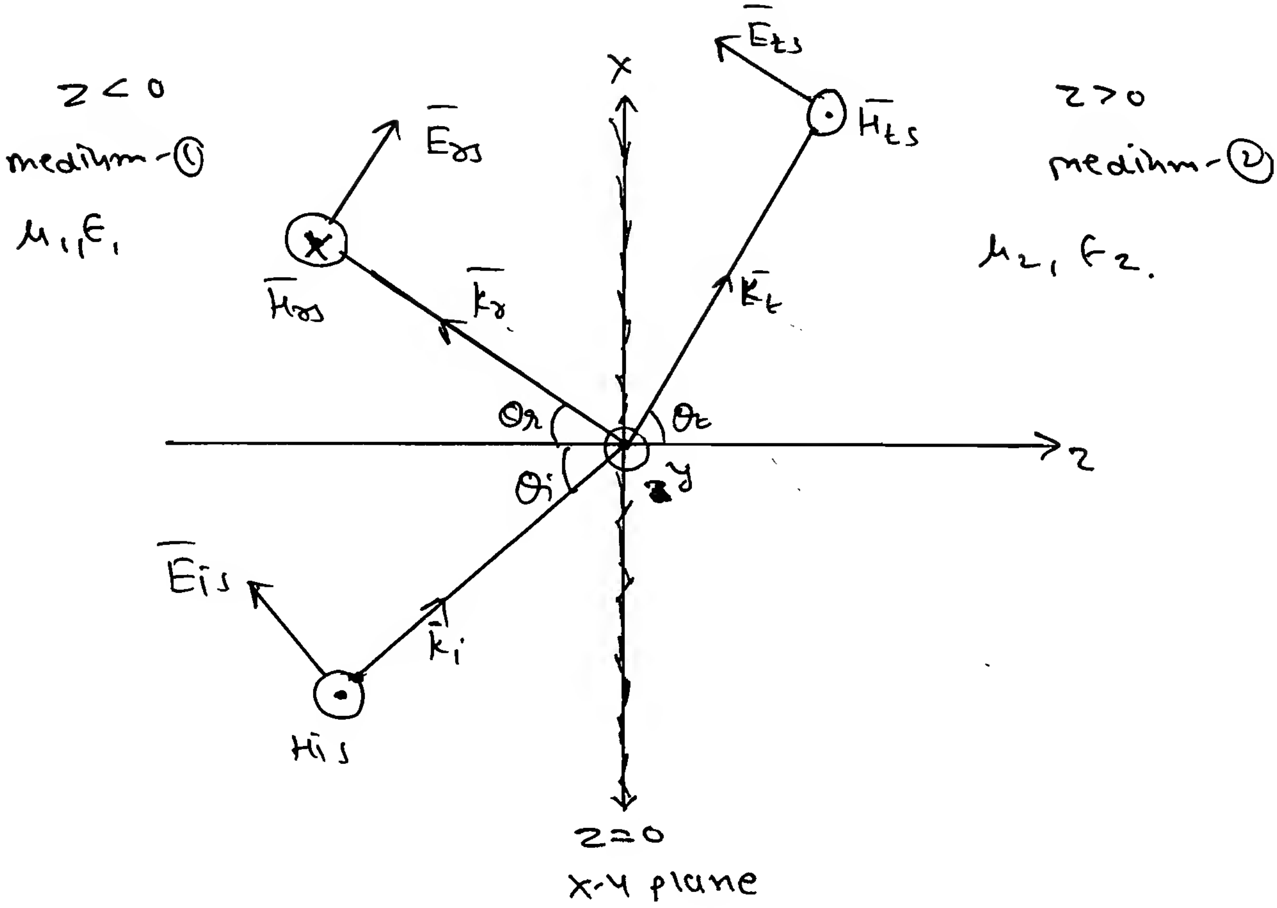
③ $\overline{E_{ts}} = \frac{16}{5}\, e^{-j\beta_2 y}\, \hat{a}_z$

$\overline{H_{ts}} = -\frac{1}{25\pi}\, e^{-j\beta_2 y}\, \hat{a}_x$

$\overline{P_{avg}} = \frac{1}{2}\, \overline{E_{ts}} \times \overline{H_{ts}^{*}} = \frac{1}{\cancel{2}} \times \frac{\cancel{16}^{8}}{5} \times \frac{-1}{25\pi}\, \hat{a}_y$

$\therefore \boxed{\overline{P_{avg}} = \frac{-8}{25\pi}\, \hat{a}_y}$ ω/m^2

* Oblique incidence : Parallel Polarization.



→ By Snell's law,

$$\theta_i = \theta_r.$$

$$\therefore \quad \frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}$$

→ figure shows that an interface is define by z = 0, z<0 is medium -① and z>0 is medium -②.

→ The wave is progressing from medium-① to medium-②.

→ If the vector corresponds to the direction of the propogation makes an angle with the unit vector normal to the interface is

oblique incidence.

→ $\bar{k_i}$, $\bar{k_r}$ and $\bar{k_t}$ are the vector corresponds to direction of propagation of incident, reflected and transmitted waves. These are making angle θ_i, θ_r and θ_t respectively.

* <u>plane of Incidence:</u>

→ It is that plane on which $\bar{k_i}$, $\bar{k_r}$, and $\bar{k_t}$ and the unit vector normal to the interface ($\hat{a_n}$) are lies on this plane.

→ As shown in figure all these are lieing on z-x plane. Therefore z-x plane is called plane of incidence.

→ The electric field vector is $\parallel$ to plane of incidence i.e. $\parallel$ to z-x plane and hence this case is said to be parallel polarization.

(If $\theta_i = 0$ then the case is said to be normal incidence).

$$\rightarrow \boxed{\Gamma_{11} = \frac{n_2 \cos\theta_t - n_1 \cos\theta_i}{n_2 \cos\theta_t + n_1 \cos\theta_i}.}$$

$$\rightarrow \boxed{T_{11} = 1 + \Gamma_{11}.}$$

* Breswer Angle: (θ_{B11})

→ It it that perticular angle of incidence for which no reflection take place.

→

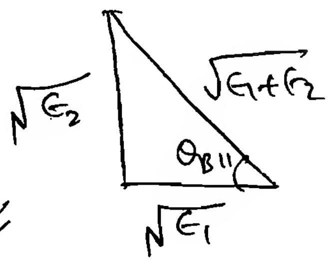
$$\therefore \boxed{\sin\theta_{11} = \sqrt{\frac{1 - \frac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2}}{1 - \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}}.}$$

for non-magnetic media $\mu_1 = \mu_2 = \mu_0$.

$$\therefore \sin\theta_{11} = \sqrt{\frac{1 - \frac{\epsilon_1}{\epsilon_2}}{1 - \left(\frac{\epsilon_1}{\epsilon_2}\right)^2}}.$$

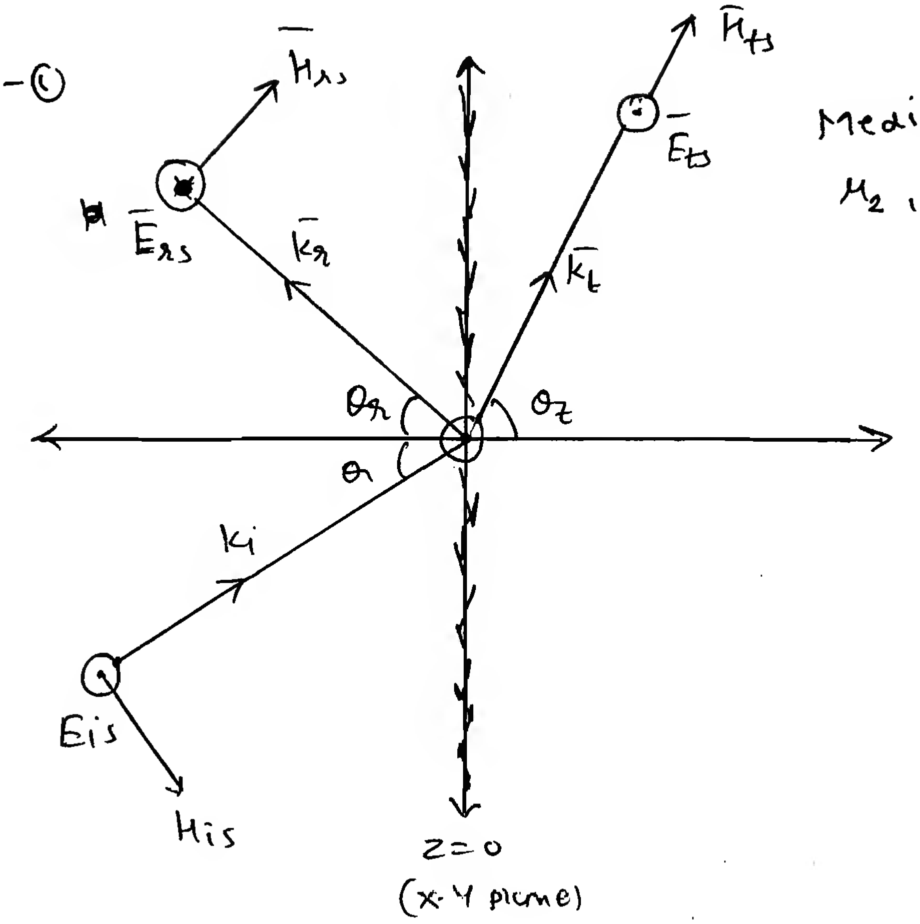
$$\therefore \sin\theta_{11} = \sqrt{\frac{\epsilon_2 - \epsilon_1}{(\epsilon_2 - \epsilon_1)^2} \times \epsilon_2}$$

$$\therefore \sin\theta_{B11} = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}.$$



(or) $\boxed{\tan\theta_{B11} = \sqrt{\epsilon_2 / \epsilon_1}}$

* Oblique Incidence: (Perpendicular Polarization)



→ $$\boxed{\Gamma_\perp = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}}$$

$$\boxed{T_\perp = 1 + \Gamma_\perp}$$

→ Electric field is perpendicular to the plane of incidence.

* Brewster Angle: 'θ_B'

$$\therefore \boxed{\sin\theta_{B\perp} = \sqrt{\frac{1 - \frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}}}$$

→ For non-magnetic media $\mu_1 = \mu_2 = \mu_0$

$\sin\theta_1 \to \infty \Rightarrow$ which is impossible.

$\therefore$ θ_B does not occure for non-magnetic media for $\perp^{er}$ Polarization.

→ In general an EM wave can be represented as

$$\therefore \boxed{\bar{E} = \text{Re}\left[\bar{E}_{r0}\, e^{j(\omega t - \bar{k}\cdot\bar{r})} \right]}$$

→ $\bar{r}$ = Radius vector (or) position vector.

$\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$.

$\bar{K}$ = Propagation vector (or) wave number vector.

→ $|\bar{K}|$ is related to β.

$\beta = \omega\sqrt{\mu\epsilon}$.

$$\boxed{|\bar{K}| = \sqrt{K_x^2 + K_y^2 + K_z^2} = \beta.}$$

* $\bar{E}$, $\bar{H}$ and $\bar{k}$ are mutually orthogonal to each other.

* $\bar{E}$, $\bar{H}$ lies in a plane defined by $\bar{K}\cdot\bar{r}$

$\bar{K}\cdot\bar{r}$ = constant.

$$\left.\begin{array}{l}\bar{k}\cdot\bar{E}=0\\ \bar{k}\cdot\bar{H}=0\\ \bar{E}\cdot\bar{H}=0\end{array}\right\}$$ mutually orthogonal.

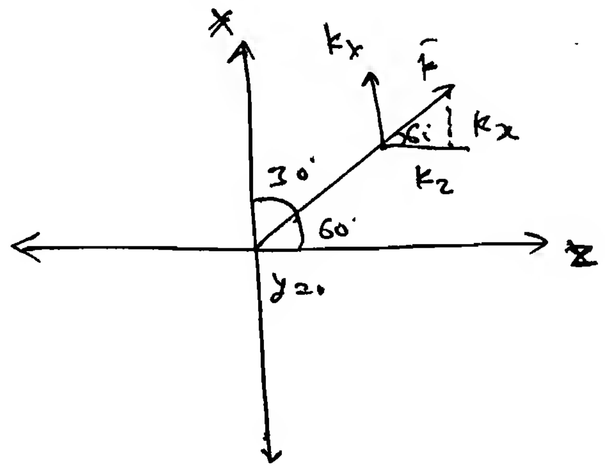
→ Further we write

$$\bar{k}\times\bar{E}=\omega\mu\bar{H}$$

$$\bar{k}\times\bar{H}=\omega\epsilon\bar{E}.$$

Ex-1 An EM waves is propagating in the free space making an angle 30° into +ve x-axis and 60° with +ve z-axis. and 90° with the +ve y-axis. Find the the vector corresponds to the direction of propagation. assume $\beta=\frac{2\pi}{\lambda}$ and also give general representation for find $\bar{E}$.

Ans:



→ $\bar{k}=k_x\hat{a}_x+k_y\hat{a}_y.$

$\therefore|\bar{k}|=\sqrt{k_x^2+k_y^2}=\beta.$

$k_x = |\bar{k}| \sin 60°$

$\therefore k_x = \beta \times \sqrt{3}/2 = \sqrt{3}/2 \times \frac{2\pi}{\lambda} = \frac{\sqrt{3}\pi}{\lambda}.$

$\therefore k_y = \beta \cos 60° = \beta \times \frac{1}{2} = \beta/2 = \frac{\lambda}{2\pi} \times \pi = \lambda/2.$

$\therefore \boxed{\bar{k} = \frac{\sqrt{3}\pi}{\lambda} \hat{a}_x + \frac{\pi}{\lambda} \hat{a}_z.}$

$\therefore \bar{E} = Re [e^{-j(\omega t - \bar{k}.\bar{r})}$

$\therefore \boxed{\bar{E} = Re [e^{-j(\omega t - \frac{\sqrt{3}\pi x}{\lambda} - \frac{\pi}{\lambda} z)}.}$

Ex-2 Interface is defined by $z=0$, $z<0$ is free space (μ_0, ϵ_0) and $z>0$ is medium-② and is characterised by $\mu_2 = \mu_0$, $\epsilon_2 = 4\epsilon_0$. A wave incidence upon the interface whose whose electric field is given by $8\cos(\omega t - 3x - 4z)\hat{a}_y$ V/m. Find angle of incidence, angle of reflection and angle of transmission, type of polarization, Γ, T, $\bar{k}_r$, $\bar{k}_t$, $\bar{k}_i$, $\bar{E}_r$, $\bar{E}_t$.

Ans:

$\bar{E}_i = 8\cos(\omega t - 3x - 4z)\hat{a}_y.$

$\therefore k_x = 3, \quad k_y = 4.$

$\therefore \beta_1 = \sqrt{k_x^2 + k_y^2}$

$\therefore \quad \beta_1 = \sqrt{9+16} = 5.$

$\therefore \quad \beta_1 = \omega\sqrt{\mu\epsilon}.$

$\therefore \quad \beta_1 = \omega\sqrt{\mu\epsilon}.$

$\therefore \quad \eta_i = \sqrt{\frac{\mu}{\epsilon}}.$

$\therefore \quad \eta_i = \sqrt{\frac{\mu_0}{\epsilon_0}}$

$\therefore \boxed{\eta_i = 120\pi}\ \Omega$

$\therefore \boxed{\beta_t = 10}$

$\eta_t = \sqrt{\frac{\mu_0}{4\epsilon_0}}$

$\eta_t = \frac{120\pi}{2}$

$\boxed{\eta_t = 60\pi}\ \Omega$

$\beta_i = \omega\sqrt{\mu_0\epsilon_0}$

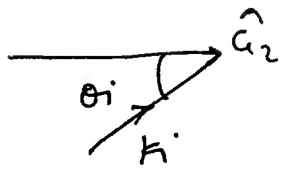
$\therefore \quad \beta_t = \omega\sqrt{\mu_2\epsilon_2}$

$\therefore \quad \beta_t = \omega\sqrt{\mu_0 4\epsilon_0}$

$\beta_t = 2\beta_i$

→ The unit vector normal to the interface $\hat{a}_n = \hat{a}_z$, the vector corresponds to direction of propagation of incident wave is $\bar{k}_i = 3\hat{a}_x + 4\hat{a}_z$ and lies on z-x plane.

→ The unit vector normal to the interface and $\bar{k}_i$ lies in z-x plane and hence z-x plane is said to be a plane of incidence given electric field has $\hat{a}_y$ component and hence the wave is said to be ~~vertically~~ perpendicular polarized.



$\therefore \quad \bar{K}_i / F \quad \bar{k}_i \cdot \hat{a}_z = |\bar{k}_i| \cdot |\hat{a}_z| \cos\theta_i$

$\therefore \cos\theta_i = \dfrac{4}{5 \times 1}$.

$\therefore \theta_i = \cos^{-1}\left(\frac{4}{5}\right)$.

$\therefore \boxed{\theta_i = 36.87^\circ}$

$\therefore$ By Snell's Law:

$\therefore \boxed{\theta_i = \theta_r = 36.87^\circ}$

$\therefore \dfrac{\sin\theta_t}{\sin\theta_i} = \sqrt{\dfrac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}$.

$\therefore \sin\theta_t = \frac{1}{2} \times \sin\theta_i$

$\bar{k}_r = -4\hat{a}_z + 3\hat{a}_x$

$\therefore \sin\theta_t = 0.3$

$\therefore \boxed{\theta_t = 17.46^\circ}$

$\therefore \bar{k}_r = -k_x \hat{a}_x + k_z \hat{a}_z$.

$\therefore k_x = \beta_i \sin\theta_i$

$\therefore k_x = 5 \sin 36.87^\circ = 3$.

$k_z = 5 \cos 36.87^\circ$

$\therefore \boxed{k_z = 4}$.

$\therefore \bar{k}_t = k_{tz} \hat{a}_z + k_{tx} \hat{a}_x$

$\therefore \bar{k}_t = \beta_2 \cos\theta_t \hat{a}_z + \beta_2 \sin\theta_t \hat{a}_x$

$\therefore \bar{k}_t = \cancel{\text{rox}}$

$$\boxed{\bar{k}_t = 9.5\,\hat{a}_z + 3.\,\hat{a}_x}$$

$$\therefore \quad \Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}.$$

$$\therefore \quad \Gamma_\perp = \frac{60\pi \cos 36.87^\circ - 120\pi \cos 17.46^\circ}{60\pi \cos 36.87^\circ + 120\pi \cos 17.46^\circ}$$

$$= \frac{150 - 359.44}{150 + 359.44}$$

$$\therefore \boxed{\Gamma_\perp = -0.411}$$

$$T_\perp = 1 + \Gamma_\perp$$

$$\boxed{T_\perp = 1.411}$$

$$\therefore \quad E_{ro} = \Gamma_\perp E_{io}$$

$$\therefore \quad E_{ro} = -0.411\, E_{io} = -3.3.$$

$$\therefore \quad \boxed{\bar{E}_r = -3.3 \cos(\omega t + \cancel{4z} - 3x)\,\hat{a}_y \text{ V/m.}}$$

$$\therefore \quad E_{to} = T_\perp E_{io} = 11.3.$$

$$\therefore \quad \boxed{\bar{E}_t = 11.3 \cos(\omega t - 9.5\hat{a}_z - 3\hat{a}_x)\,\hat{a}_y \text{ V/m.}}$$

☆ Wave guides:

→ waveguides (or) transmission lines which are used at microwaves frequencies. These are cylindrical in structure. The waveguides prefered cross section of the are Rectangular, circular or eliptical. No other cross sections are prefered because there are no advantages found with the other cross sections.

→ Waveguides are the examples for the wave propagation through a bounded medium.

→ ~~The~~ ~~medium~~ Wave guides has conductor boundaries.

→ The medium betⁿ the conductor boundaries is non-conducting. It may be filled with dielectric material.

→ For example air filled waveguide.

→ The energy progresses along the length of the waveguide. to investigate electromagnetic field behaviour inside the wave-guide. Maxwell's eqⁿs and the wave eqⁿs are solved subjected to the boundary

conditions (or) electrical & magnetic fields across the conductor boundaries.

→ we know that tangential component of electric field and normal component of magnetic fields vanishes across 4 conductor boundaries.

→ We consider rectangular waveguides.

→ Rectangular waveguides means its cross section is Rectangular.

⇒

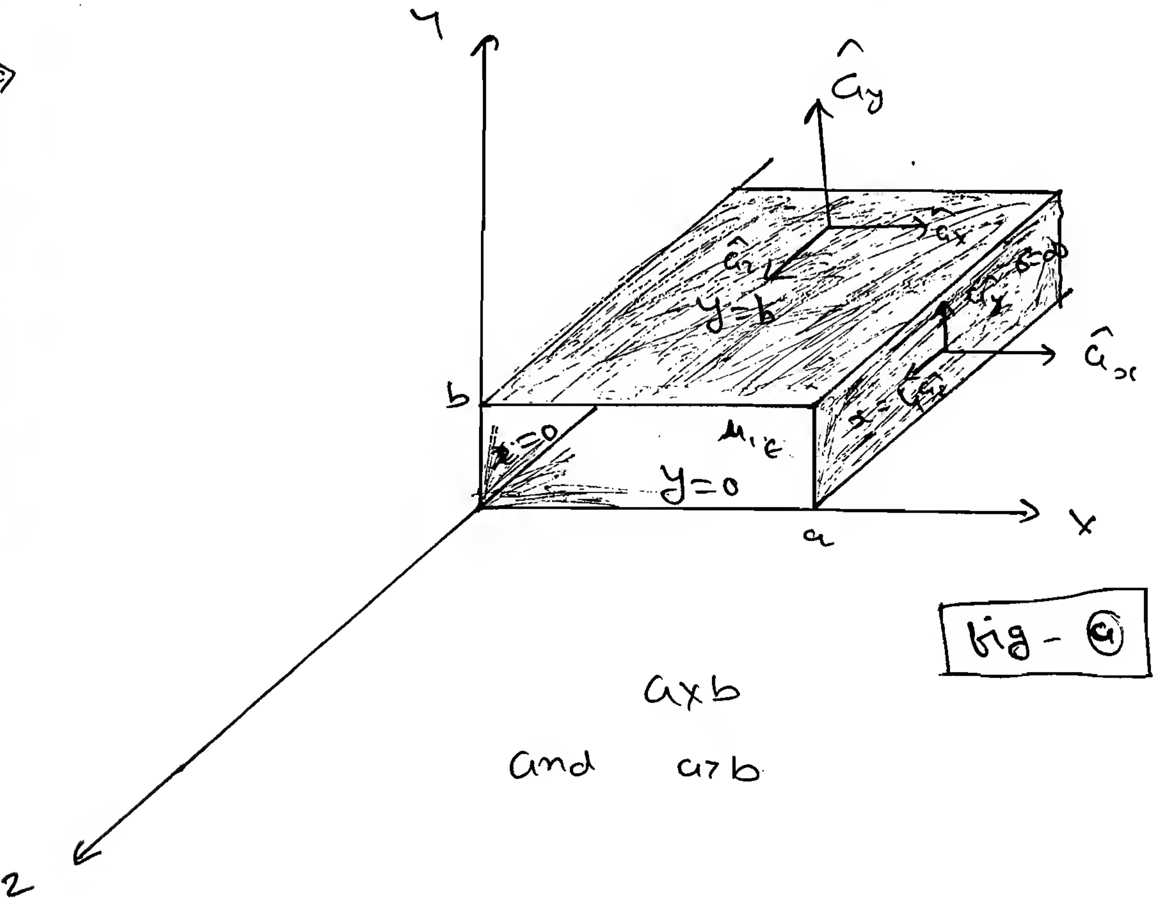


fig - ⓐ

a×b

and a>b

→ In general,

$$\bar{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z .$$

$$\bar{H} = H_x \hat{a}_x + H_y \hat{a}_y + H_z \hat{a}_z .$$

→ Fig. Shows a rectangular waveguide with the cross section ~~direction~~ dimension $a \times b$ and $a > b$.

→ There exist four conducting plane which are located at $x=0$, $x=a$, $y=0$, $y=b$.

→ This conducting planes are assumed to have infinite conductivity & medium betn the conducting plane is linear, homogenius, isotropic, Charge free, Non-conducting. For example in a air field waveguide.

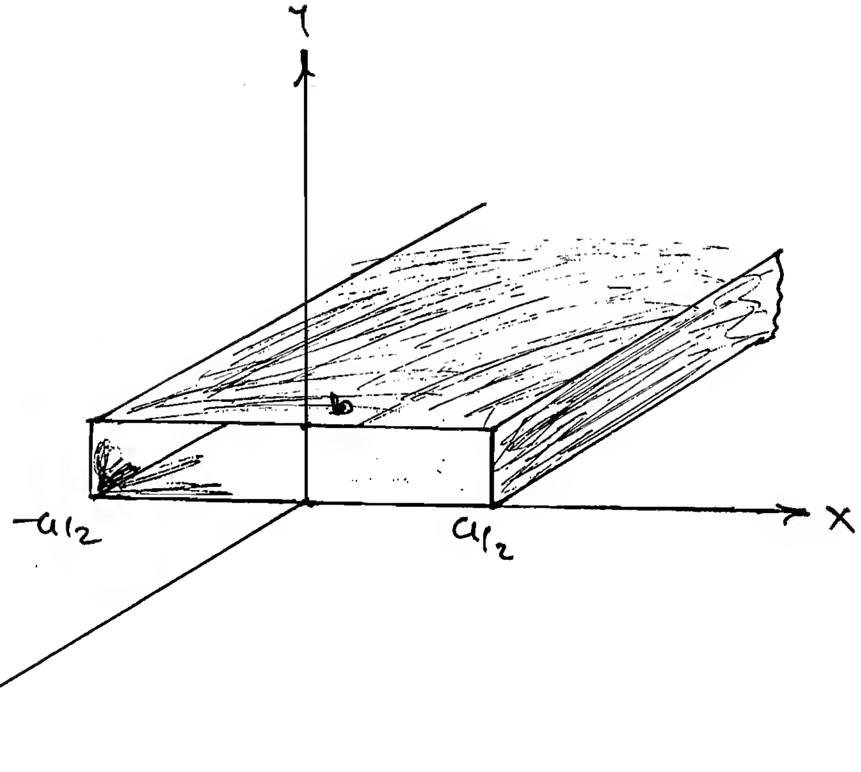
→ There exist conductor interfaces which are located at $x=0$, $x=a$, $y=0$, $y=b$.

→ Tangential component of electric fields & normal component of magnetic field vanishes across the conductor interfaces.

→ at $\boxed{\begin{array}{l} x=0,\ x=a \\ E_y=0,\ E_z=0,\ H_x=0. \end{array}}$

→ at $\boxed{\begin{array}{l} y=0,\ y=b. \\ E_x=0,\ E_z=0,\ H_y=0. \end{array}}$

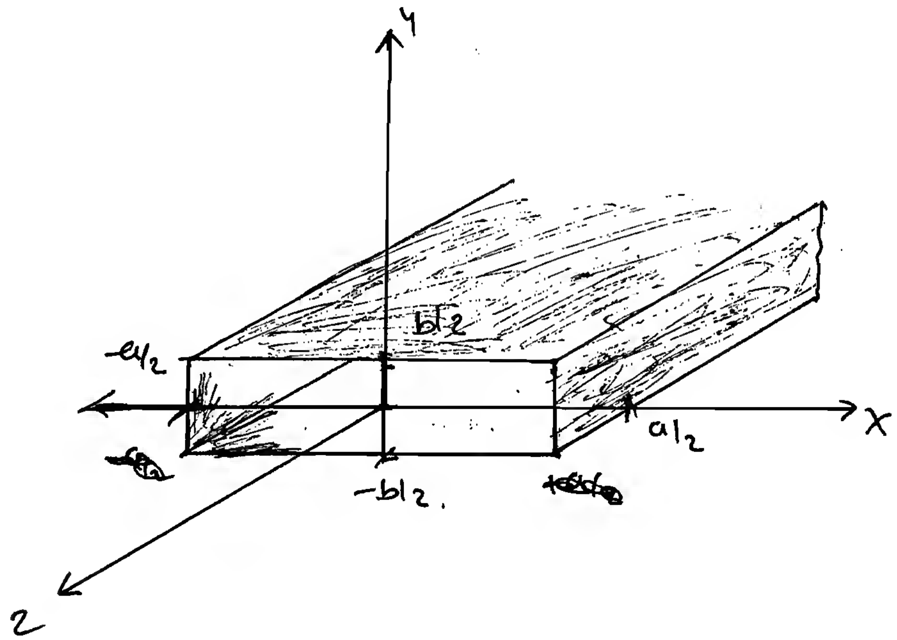
Fig- ⓑ



→ $\boxed{\text{at} \quad x = \pm a/2, \quad E_y = 0, \; E_z = 0, \; H_x = 0}$

→ $\boxed{\text{at} \quad y = 0 \;\&\; y = b. \quad \therefore \; E_x = 0, \; E_z = 0, \; H_y = 0.}$

Fig- ⓒ



→ $\boxed{\begin{array}{l} \text{at} \quad x = \pm a/2. \\ \therefore \quad E_y = 0, \quad E_z = 0, \quad H_x = 0 \end{array}}$

→ $\boxed{\begin{array}{l} \text{at} \quad y = \pm b/2. \\ \therefore \quad E_x = 0, \quad E_z = 0, \quad H_y = 0. \end{array}}$

→ We consider fig-(a) for our analysis.

→ As shown in the figure -(a) there exist 4- conducting planes. which are located at $x=0$, $x=a$, $y=0$, $y=b$.

→ This conducting planes are assume to have infinite conductivity and the medium betn conducting planes is linear, homogenius, isotropic, charge free and non-conducting.

→ Writting the maxwell's eqn's for the medium assumed betn the conducting planes.

→ ① $\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$.

② $\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t}$ (Non conduing medium is assumed $\sigma=0$).

③ $\nabla \cdot \bar{D} = 0$. ($\because$ Charge free medium is assumed $\rho_v=0$).

$\rightarrow \quad \epsilon \nabla \cdot \bar{E} = 0$

$\nabla \cdot \bar{E} = 0.$ (homo genius ~~eqn~~ medium).

(4) $\nabla \cdot \bar{B} = 0$

$\nabla \cdot \bar{H} = 0$ (homo genius medium).

→ Taking curl on eqn-① both the side,

$$\nabla \times \nabla \times \bar{E} = -\mu \nabla \times \frac{\partial \bar{E}}{\partial t}.$$

$$\therefore \quad \underbrace{\nabla(\nabla \cdot \bar{E})}_{0} - \nabla^2 \bar{E} = -\mu \, \partial/\partial t \, \underbrace{(\nabla \times \bar{E})}_{\sigma \frac{\partial \bar{E}}{\partial t}}.$$

$$\therefore \quad \nabla^2 \bar{E} = \mu \epsilon \frac{\partial \bar{E}}{\partial t}.$$

$$\text{llel} \quad \nabla^2 \bar{H} = \mu \epsilon \frac{\partial \bar{H}}{\partial t}.$$

Vector wave eqn.

→ Expanding wave eqn, in the cartesian co-ordinates because the geometry is well suited for cartesian co-ordinates.

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}.$$

$$\frac{\partial^2 \bar{H}}{\partial x^2} + \frac{\partial^2 \bar{H}}{\partial y^2} + \frac{\partial^2 \bar{H}}{\partial z^2} = \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2}.$$

$\rightarrow \quad \bar{E} = \mathrm{Re}\,[\bar{E}_s e^{j\omega t}].$

Writing the above eqn in phasor form.

$$\frac{\partial^2 \bar{E}_s}{\partial x^2} + \frac{\partial^2 \bar{E}_s}{\partial y^2} + \frac{\partial^2 \bar{E}_s}{\partial z^2} = -\omega^2 \mu \epsilon E_s.$$

similarly $$\frac{\partial^2 \bar{H}_s}{\partial x^2} + \frac{\partial^2 \bar{H}_s}{\partial y^2} + \frac{\partial^2 \bar{H}_s}{\partial z^2} = -\omega^2 \mu \epsilon H_s.$$

→ When a wave is propagating along z-direction in the unbounded medium. we have conclude that the partial variations of any field component w.r.t. x & y vanishing.

→ Where as in the case of fig-ⓐ if we assume the energy is being progressing along the length of the wave guide i.e. along z-direction, since there are conducting walls which are located at $x=0$, $x=a$, $y=0$ and at $y=b$, then it is not possible to assume partial variation of any field component w.r.t. to x & y to be zero.

NOTE:

→ we have assumed that energy is being progressing along z-direction the variations along z can be approximated as $e^{-\bar{\gamma} z}$

where $\bar{\gamma} = \bar{\alpha} + j\bar{\beta}$.

→ $\bar{\gamma}$ is not a vector quantity to distinguish ~~constant~~ $\bar{\gamma}$ of the waves are propagating in waveguide $\bar{\gamma}$ is used.

→ $$\bar{E}_s(x,y,z) = \bar{E}_{so}(x,y) \cdot e^{-\bar{\gamma} z}.$$

$$\frac{\partial \bar{E}_s}{\partial z} = -\bar{\gamma}\bar{E}_s$$

$$\frac{\partial^2 \bar{E}_s}{\partial z^2} = \bar{\gamma}^2 \bar{E}_s.$$

→ The wave eqs. can be written as

$$\left.\begin{aligned} &\frac{\partial^2 \bar{E}_s}{\partial x^2} + \frac{\partial^2 \bar{E}_s}{\partial y^2} + \bar{\gamma}^2 \bar{E}_s + \omega^2 \mu \epsilon \bar{E}_s = 0 \\ &\text{lly} \quad \frac{\partial^2 \bar{H}_s}{\partial x^2} + \frac{\partial^2 \bar{H}_s}{\partial y^2} + h^2 \bar{H}_s = 0 \end{aligned}\right\} \text{These are } 2^{nd} \text{ order 2-DIM PDEs.}$$

where, $h^2 = \gamma^2 + \omega^2 \mu\epsilon$.

→ The above second order PDE can be split into two simple 2^{nd} order differential eqns. They will be in the form of harmonic eqns. Soln of ~~an~~ harmonic eqn may take either sine or cosine or exponent. We consider sine and cosine forms.

→ First two Maxwell eqn in phasor form:

$$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$$

&

$$\nabla \times \bar{H}_s = j\omega\epsilon \bar{E}_s$$

set-2 set-1

$$\frac{\partial E_{zs}}{\partial y} - \underbrace{\frac{\partial E_{ys}}{\partial z}}_{-\bar{\gamma} E_{ys}} = -j\omega\mu H_{xs} \qquad \frac{\partial H_{zs}}{\partial y} - \underbrace{\frac{\partial H_{ys}}{\partial z}}_{-\bar{\gamma} H_{ys}} = j\omega\epsilon E_{xs}$$

$$\underbrace{\frac{\partial E_{xs}}{\partial z}}_{-\bar{\gamma} E_{xs}} - \frac{\partial E_{zs}}{\partial x} = -j\omega\mu H_{ys} \qquad \underbrace{\frac{\partial H_{xs}}{\partial z}}_{-\bar{\gamma} H_{xs}} - \frac{\partial H_{zs}}{\partial x} = j\omega\epsilon E_{ys}.$$

$$\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} = -j\omega\mu H_{zs} \qquad \frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} = j\omega\epsilon E_{zs}.$$

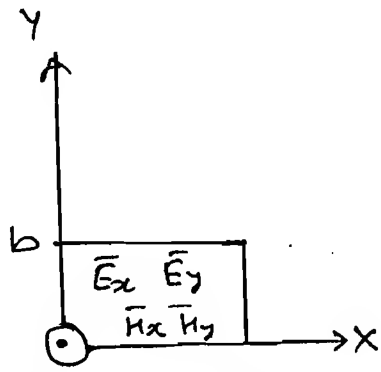
→ Using set ① — E_{xs} → can be written in forms of E_{zs}, H_{zs}.
— H_{ys} → can be written in forms of E_{zs}, H_{zs}.

→ Using set ② — E_{ys} → can be written in forms of E_{zs}, H_{zs}
— H_{xs} → can be written in forms of E_{zs}, H_{zs}.

→ $$\bar{E}_s = \bar{E}_{s0}(x,y)\, e^{-\bar{\gamma} z}$$

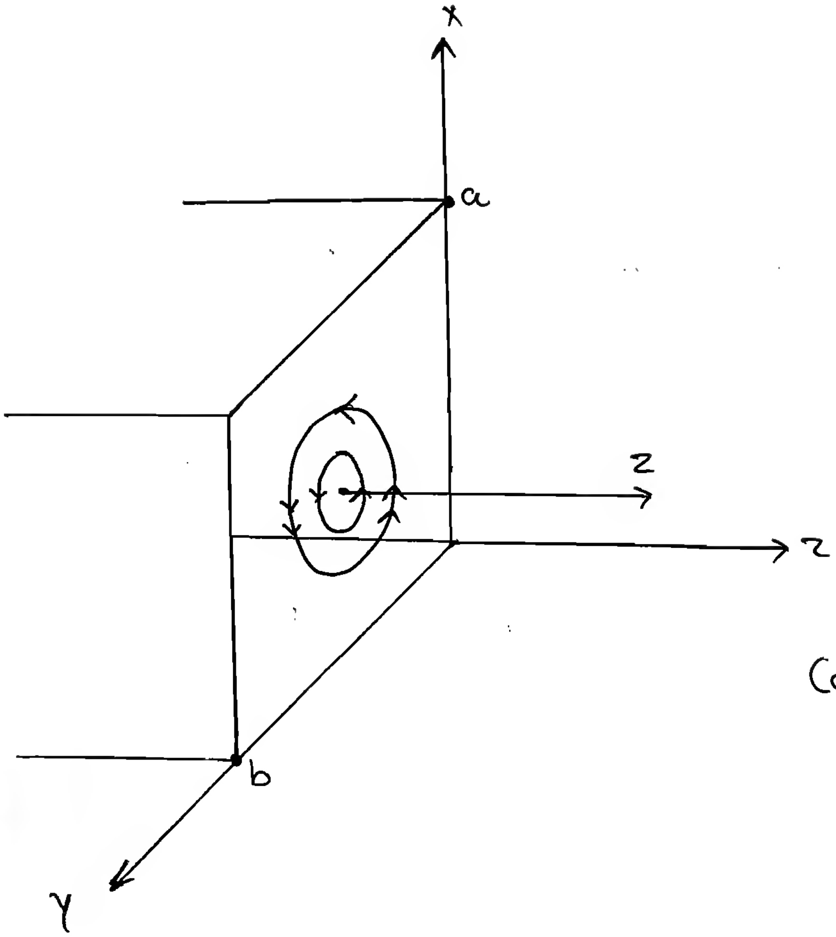
$$\bar{E}_{ys} = E_{ys0}(x,y)\, e^{-\bar{\gamma} z}$$

$$\frac{\partial E_{ys}}{\partial z} = -\bar{\gamma} E_{ys}$$



~~TEM to z~~

~~$E_z = 0$, $H_z = 0$~~



axial current

Conduction

~~$J_{Dz} = \epsilon \frac{\partial E_z}{\partial t}$ displacement~~

→ With reference to fig-①, we have assumed that the energy is been progressing along z direction the transverse plane would be xy plane. The field component which lies in the transverse plane are E_x, E_y, H_x, H_y

→ The field component along the direction of progation of energy are E_z & H_z.

→ As shown above the transverse field components have been represented interms of the field components along the directions of propagation of energy (i.e.) E_x, E_y, H_x, H_y have been represented in terms of

E_z & H_z. To have TEM wave propogation to z, then E_z and H_z must be zero.

→ If these are zero, no field component is existing inside the waveguide.

→ We can conclude that TEM wave propogation is impossible to exist through any cylindrical waveguide system of any cross section with no central conductor.

Waves in the waveguide

m, n : integers

→ TM_{mn} waves

→ Transverse magnetic waves.

→ $H_z = 0$

→ (i.e) magnetic field lies entirely in the transverse plane.

→ These are called E-waves

→ TE_{mn} waves

→ Transverse Electric waves

→ $E_z = 0$

(i.e) Electric field lies entirely in the transverse plane.

→ These are also called H-waves.

→ $F_1(x)$ $F_2(y)$ $F_3(z)$ $F_4(t)$

Any field Components is a product of four independent functions

$$\cos\left(\frac{m\pi}{a}\right)x \rightarrow \cos\left(\frac{n\pi}{b}\right)y \rightarrow e^{-\gamma z} \rightarrow e^{j\omega t}$$

(or) (or)

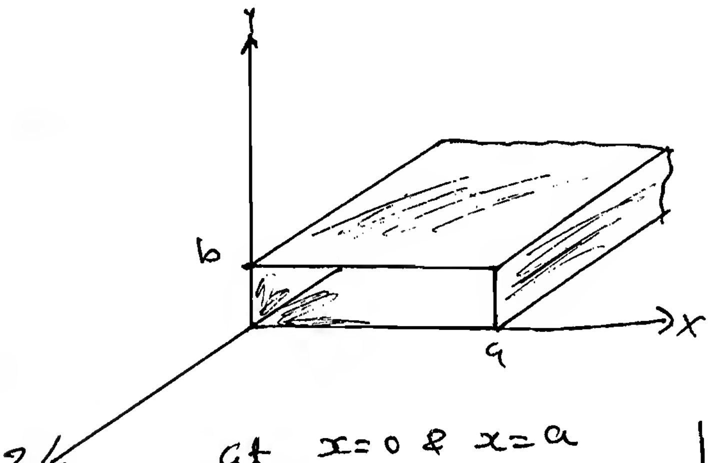
$$\sin\left(\frac{m\pi}{a}\right)x \rightarrow \sin\left(\frac{n\pi}{b}\right)y$$

$F_1 \rightarrow$ function of 'x' alone.

$F_2 \rightarrow$ Function of y alone

$F_3 \rightarrow$ Function of z alone.

$F_4 \rightarrow$ function of t alone.



at $x=0$ & $x=a$

$E_y=0$, $E_z=0$

$H_x=0$

at $y=0$, $y=b$.

$E_x=0$, $E_z=0$.

$H_y=0$

* TM_{mn} Waves:

→ $E_{xs} = E_{x0}\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}$. $H_{xs} = -\frac{E_{ys}}{\eta_{TMmn}}$.

→ $E_{ys} = E_{y0} \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}$, $H_{ys} = \frac{E_{xs}}{\eta_{TMmn}}$.

→ $E_{zs} = E_{z0} \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z}$, $H_{zs}=0$.

$\longrightarrow (+z)$

$$\frac{E_x}{H_y} = \eta_{TMmn} = \frac{-E_y}{H_x}$$

→ ~~η_{TEmn}~~

→ $\eta_{TM_{mn}}$ is the Characteristic wave independence of TM_{mn} waves.

* **TEmn Waves.**

→ $E_{xs} = E_{xo} \cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right) \cdot e^{-\bar{\gamma} z}$, $H_{xs} = \frac{-E_{ys}}{\eta_{TE_{mn}}}$.

→ $E_{ys} = E_{yo} \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right) \cdot e^{-\bar{\gamma} z}$, $H_{ys} = \frac{E_{xs}}{\eta_{TE_{mn}}}$.

$E_{zs} = 0$, $H_{zs} = H_{zo} \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y e^{-\gamma z}$.

$\longrightarrow (+z)$

$$\frac{E_x}{H_y} = \eta_{TE_{mn}} = -\frac{E_y}{H_x}.$$

→ $\eta_{TE_{mn}}$ is the Characteristic wave impedence of TE_{mn} waves.

* **Characteristics of TEmn waves & TMmn waves.**

$$\bar{\gamma}^2 + \omega^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2.$$

$$\therefore \boxed{\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon.}}$$

We have known → $\boxed{\bar{\gamma} = \bar{\alpha} + j\bar{\beta}}$

m, n : integers (mode).

a, b : Cross sectional dimensions.

μ, ϵ : Medium Properties (Linear, Homogeneous, Isotropic).

$\underline{\omega = 2\pi f}$ ← freq. of the wave.

$$\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}.$$

→ If $\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 > \omega^2 \mu \epsilon \Rightarrow \bar{\gamma}$ is purely real

$\Rightarrow$ ~~No~~ $\bar{\beta} = 0 \Rightarrow$ No propogation takes place

→ If $\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 < \omega^2 \mu \epsilon \Rightarrow \bar{\gamma}$ is purely imaginary

$\Rightarrow \alpha = 0 \Rightarrow$ ~~No~~ Propagation takes place.

→ For a given waveguide,

→ From the above we can conclude that for the Low freqw $\bar{\gamma}$ is purely real therefore propogation through the waveguide is not possible.

→ At high freq. $\bar{\gamma}$ is purely imaginary therefore propagation is allowed through the wave guide.

→ we defined a limiting freq. called cut-off freq. i.e. at $f = f_c$ (OR) $\omega = \omega_c \Rightarrow \boxed{\bar{\gamma} = 0}$

$$\omega_c^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2.$$

$$\therefore \quad \omega_c = \sqrt{\frac{1}{\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

(or)

$$\boxed{f_c = \frac{1}{2\pi\sqrt{\epsilon\mu}} \times \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{\frac{1}{2}}}$$

→ Corresponds to 'f_c' we define the cut-off wavelength such that

$$f_c \lambda_c = \frac{1}{\sqrt{\mu\epsilon}} = V_0.$$

(or)

$$\lambda_c = \frac{1}{f_c \times \sqrt{\mu\epsilon}}.$$

$$= \frac{2\pi}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}}.$$

$$\therefore \quad \boxed{\lambda_c = \frac{2}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{\frac{1}{2}}}}$$

f_c or λ_c : They purely depend upon the medium properties i.e. m, n, a, & b. (OR) physical properties of the waveguide.

→ If 'f' is the freq. of the wave and λ is the wavelength of the wave such that $f\lambda = \frac{1}{\sqrt{\mu\epsilon}} = v_0$.

∴ If $\boxed{f > f_c \text{ (or) } \lambda < \lambda_c}$ ⇒ propogation is allowed through the waveguide.

→ If $f < f_c$ (or) $\lambda > \lambda_c$ ⇒ Propogation don't allow through the wg.

→ Thus, the Waveguide is simulating the action of a High pass filter.

→ For $f > f_c$,

$$\bar{\gamma} = j\beta.$$

∴ $$\boxed{\beta = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \text{ rad/s.}}$$

∴ $$\boxed{\bar{v} = \frac{\omega}{\beta}} \text{ m/s.}$$

∴ $$\boxed{\bar{\lambda} = \frac{2\pi}{\beta}};$$

$\bar{\lambda}$ = Guide wave length. (or) wavelength measured inside the waveguide.

$$\boxed{\eta_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} \ \Omega} \qquad \boxed{\eta_{TM} = \eta\sqrt{1-(f_c/f)^2} \ \Omega}$$

where $\eta = \sqrt{\frac{\mu}{\epsilon}}$ || $\eta_{TE} \cdot \eta_{TM} = \eta^2$.

At $f = f_c \Rightarrow \bar{\beta} \to 0$

$\bar{v} \to \infty$

$\bar{\lambda} \to \infty$.

$\eta_{TE} \to \infty$

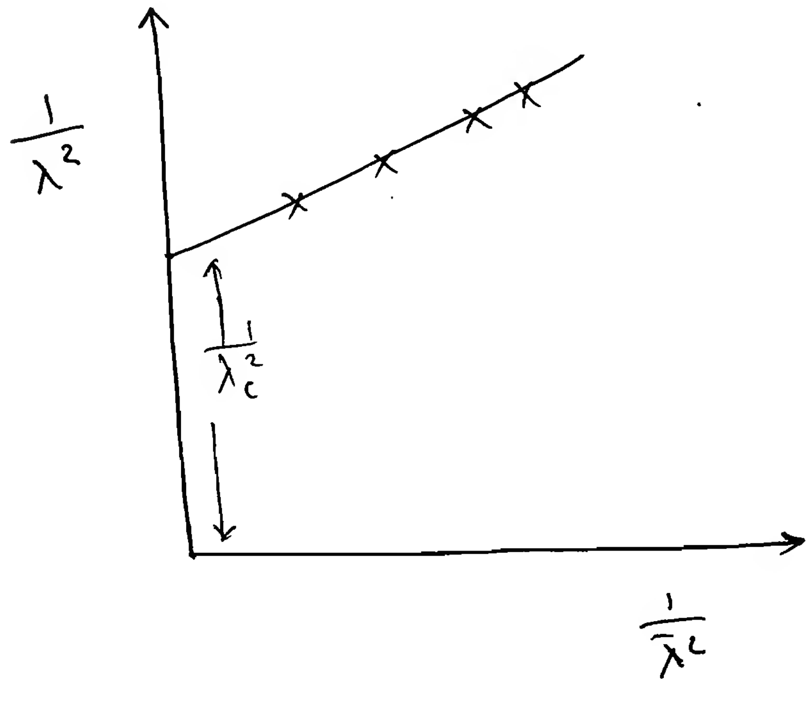
$\eta_{TM} \to 0$.

→ The relation b/w $\lambda, \lambda_c, \bar{\lambda}$

$$\frac{1}{\bar{\lambda}^2} = \frac{\bar{\beta}^2}{(2\pi)^2} = \frac{(\omega^2\mu\epsilon)}{(2\pi)^2} - \frac{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}{(2\pi)^2}$$

$$\Rightarrow \frac{1}{\bar{\lambda}^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}.$$

(or) $$\boxed{\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}}$$

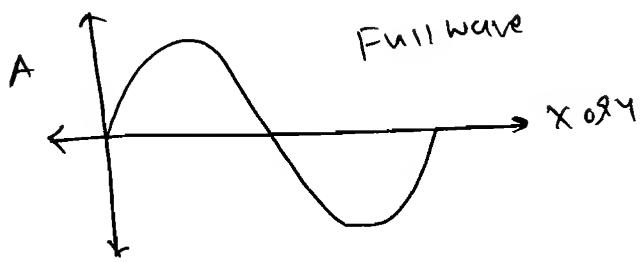


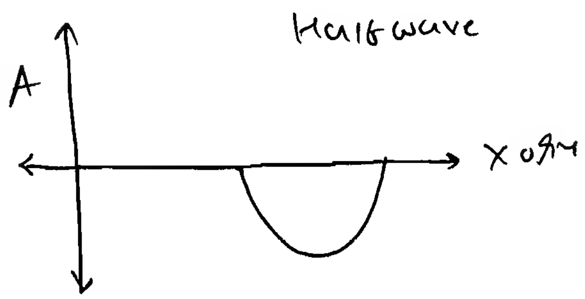
* Significance of m, n:

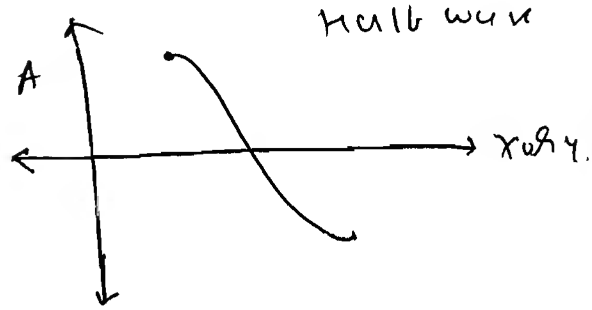
→ m: no of half field variations along x.

n: no. of half field variations along Y.

→







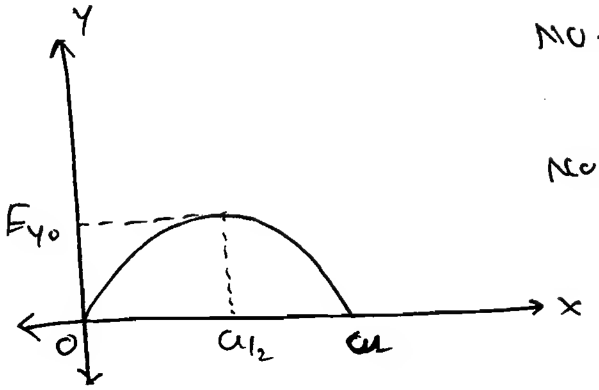
→ $m=1, \; n=0.$

TE_{10} $\boxed{f > f_c}$

- $E_{xs} = 0.$
- $E_{ys} = E_{y0} \cdot \sin\left(\frac{\pi x}{a}\right) \cdot e^{-j\beta z}.$
- $E_{zs} = 0$

$$|\bar{E}_s| = |E_{ys}| = E_{y0} \sin\left(\frac{\pi x}{a}\right).$$

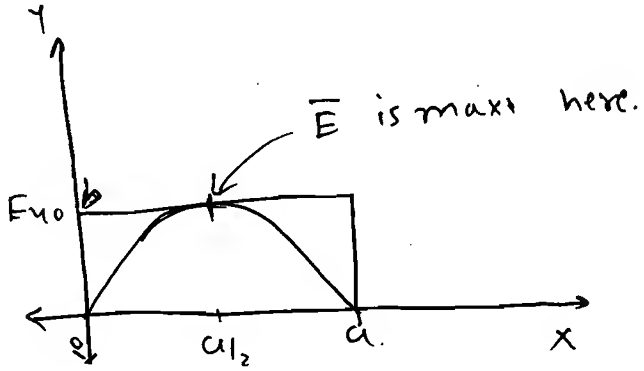
(Say at $z = 0$)



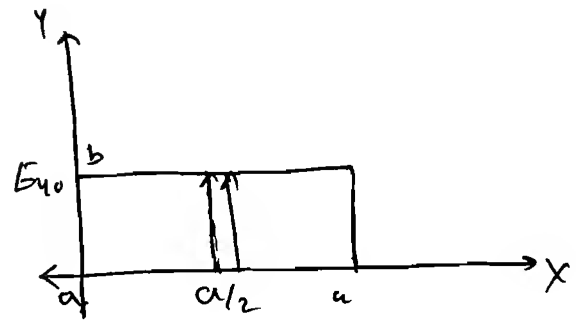
No. of half field along $x = 1$

No. of half fields along $y = 0$.

$\Downarrow$



(OR)



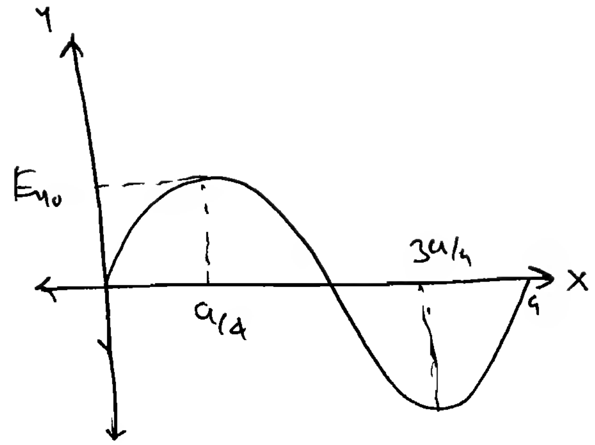
→ Let us take $m = 2$, $n = 0$.

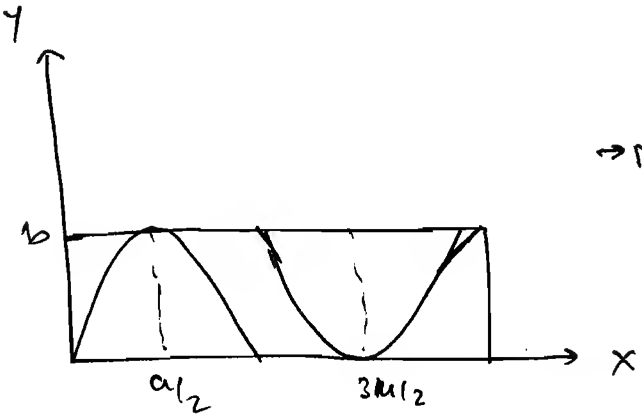
TE_{20}, $\boxed{f > f_c}$

$$\therefore E_{ys} = E_{y0} \sin\left(\frac{2\pi x}{a}\right) e^{-j\beta z}$$

$$\therefore |\bar{E}_s| = |E_{ys}| = E_{y0} \sin\left(\frac{2\pi x}{a}\right).$$

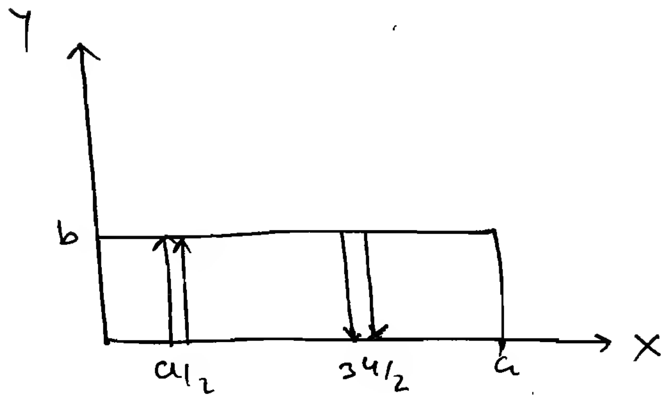
(Say at $z = 0$)





→ No. of half fields along x = 2.

→ No. of half fields along y = 0.



* Let us take $m=0, n=1$:

→ TE_{01}, $f > f_c$.

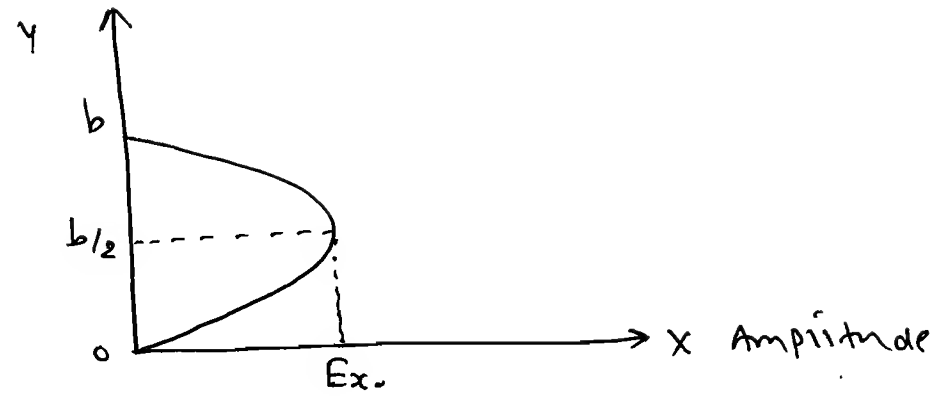
$$\therefore \bar{E}_{xs} = E_{x0} \sin \frac{\pi}{b} y \cdot e^{-j\beta z}.$$

$$\therefore |\bar{E}_s| = |\bar{E}_{xs}| = E_{x0} \sin \frac{\pi}{b} \cdot y.$$

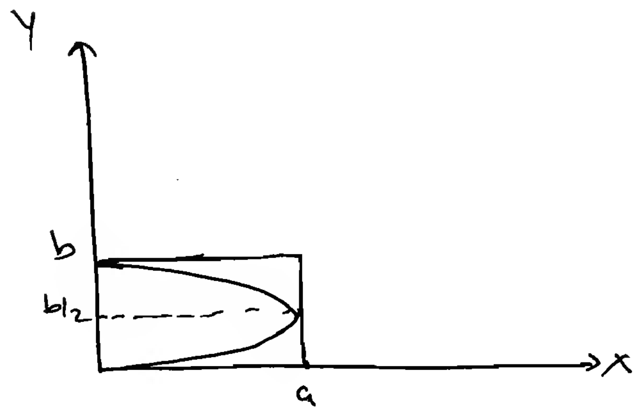
No. of half fields along $y = 1$

No. of half-fields along $x = 0$.

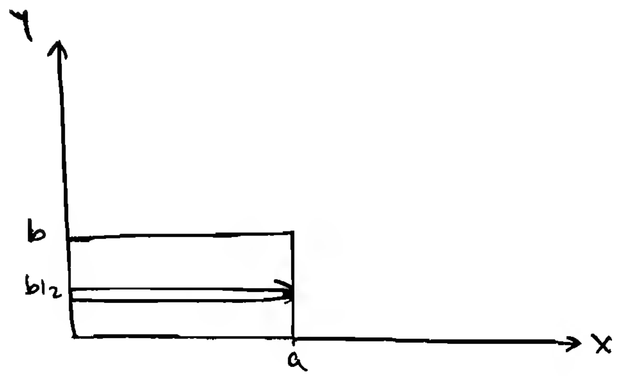
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$



* Dominant Mode:

→ It is the lowest possible propogating mode and must have lowest cut-off frequency. In the dominant mode it is possible to transfer maximum energy from source to the load.

→ In the case of the rectangular waveguide

$\boxed{TE_{10} \text{ is the dominant mode.}}$

→ In the case of TM_{mn} the lowest possible propogating mode is $\boxed{TM_{11}}$ i.e. minimum value of m, n are at least 1, 1 respectively.

Therefore, the lowest possible mode in the case of TM_{mn} is TM_{11}. This is not called dominant mode.

→ **Ex-1** A Rectangular wave guide with a cross-section dimensions 4 cm × 7 cm is air filled. The waveguide is intended to operate at the following frequencies. (i) 3000 MCPS. (ii) 5000 MCPS. What are the different modes that can be propogated through this WG at the above freq.

Ans:

$\lambda_c < \lambda$.

To have propogation $f > f_c$ (or) ~~f_c~~.

$$\therefore f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \times \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} < f.$$

$\therefore$ air filled so $\mu = \mu_0$, $\epsilon = \epsilon_0$

$$\therefore f_c = \frac{1 \times \pi}{2\pi\sqrt{\mu_0\epsilon_0}} \times \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2} < f$$

$$\therefore \frac{3 \times 10^8}{2} \times \left[\left(\frac{m}{0.04}\right)^2 + \left(\frac{n}{0.07}\right)^2\right]^{1/2} < f.$$

$$\therefore \boxed{16m^2 + 49n^2 < 31.4.}$$

Values of m	Values of n	Satisfaction
1	0	yes.
0	1	No
1	1	No

Possible mode

TE_{10}.

(ii) $f = 6000$ mcps.

$f = 6 \times 10^{9}$ Hz.

$a = 0.07$ m , $b = 0.04$ m.

→ $16m^2 + 49n^2 < 125.6$

Values of m	Values of n	Satisfaction
0	1	yes
1	0	yes.
1	1	yes.
2	0	yes.
2	1	yes.
2	2	~~yes.~~ No
0	2	~~yes.~~ No.

→ possible modes.

TE_{10}, TE_{01}, TE_{11}, TE_{20}, TE_{21}, TE_{22}, TE_{2}.

TM_{11}, TM_{21}.

* Degenerate modes:

→ If different modes are having same cut-off freq. then those modes are said to be degenerate modes.

→ In the above example TE_{11}, TM_{11} and TE_{21}, TM_{21} are Degenerate modes respectively.

* Evanscent wave.

→ Evanscent waves are the modes which can to not be propagate through the waveguide.

Ex-1 A Rectangular wave guide with a cross-section dimension $0.9'' \times 0.4''$ is air filled and is operating in the dominant mode. find cut off freq.? decide a 10 GHz wave can be propogated or not? If it is propogated calculate $\bar{\beta}$, $\bar{\lambda}$, $\eta_{TE_{10}}$ and $\bar{v}$.

Ans:

→ The expression for the avg. Power tranport through a rectangular Waveguide in the dominant mode is given by

$$\boxed{W_{avg} = \frac{E_{yo}^2}{4\eta_{TE_{10}}} ab \text{ Watts.}}$$

E_{yo}: Peak Electric field.

a, b: dimension of C/S.

$\eta_{TE_{10}}$: Characterestic wave impedence.

Ex-1 A Rectangular waveguid with c/s dimensions 2cm x 1cm is operating in the dominant mode at the freq. 30 GHz. it transport energy at the rate of 0.5 HP = 373 watts (1HP = 746 watts). What is the peak value of electric field occuring inside the waveguide.

Ans: $f = 30\,GHz = 3\times10^{10}$ Hz.

$a = 2cm = 0.02m$

$b = 1cm = 0.01m.$

$W_{avg} = 0.5HP = 373$ watts.

$$\lambda_c = \frac{2}{\left[\left(\frac{m}{a}\right)^2 + \left(n/b\right)^2\right]^{1/2}} \qquad m=1, \quad n=0$$

$$\therefore \lambda_c = 2a.$$

$$\boxed{\lambda_c = 0.04\,m.}$$

$$\therefore \lambda = \frac{v_0}{f} = \frac{3\times10^8}{3\times10^{10}} = 10^{-2} = 0.01\,m.$$

$\therefore$ $\boxed{\lambda_c > \lambda}$ so, TE$_{10}$ mode is propagated.

Now, $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\ \Omega$

$$\therefore \eta_{TE10} = \frac{\eta}{\sqrt{1-\left(f_c/f_0\right)^2}}$$

$$= \frac{\eta}{\sqrt{1-\left(\lambda_0/\lambda_c\right)^2}}$$

$$= \frac{120\pi}{\sqrt{1-\left(\frac{0.01}{0.04}\right)^2}}$$

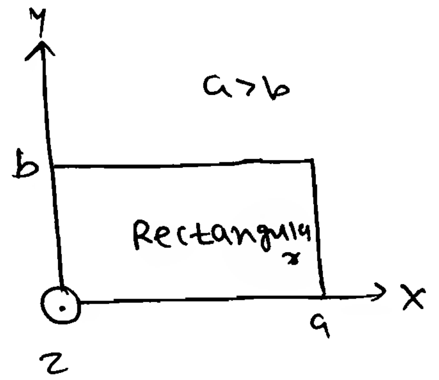
$$\therefore \eta_{TE10} = \frac{120\times2\pi}{\sqrt{3}}.$$

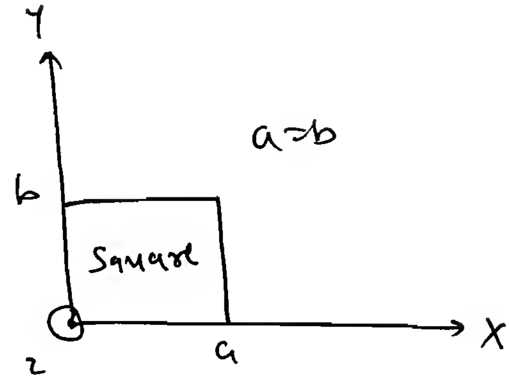
$$\boxed{\eta_{TE10} = 389\,\Omega.}$$

$$\rightarrow \quad E_{y0} = \sqrt{\frac{W_{avg}\,4\,\eta_{TE10}}{ab}}$$

$$\therefore \boxed{E_{y0} = 53\ kV/m.}$$

*





$\boxed{TE_{10}}$

$\lambda_{c10} = 2a$ $\qquad\qquad$ $\lambda_{c10} = 2a$

$\boxed{TE_{01}}$

$\lambda_{c01} = 2b$ $\qquad\qquad$ $\lambda_{c01} = 2b = 2a.$

→ The mode properties of TE_{10} and TE_{01} identically same in the case of a square waveguide whereas these are different in the rectangular waveguide. therefore different modes are not possible with square waveguides.

→ The wave propogation through the waveguide is by means of total internal reflection betⁿ the walls.

→ $$E_{ys} = E_{yo} \sin \frac{\pi x}{a} \cdot e^{-j\beta z}$$

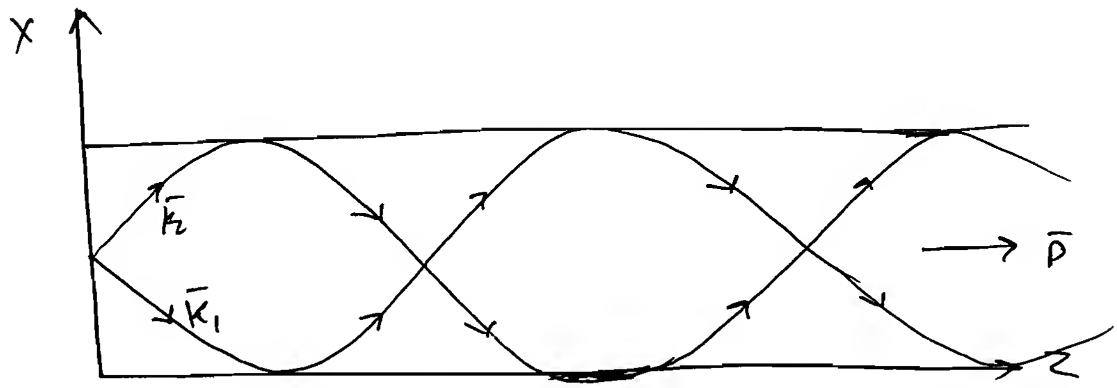
$$= E_{yo} \frac{e^{j\pi x/a} - e^{-j\pi x/a}}{2j} \cdot e^{-j\beta z}.$$

$$\therefore E_{ys} = E_{y_0}'\left[e^{j\left(\frac{\pi x}{a} - \bar{\beta} z\right)} - e^{-j\left(\frac{\pi x}{a} + \bar{\beta} z\right)} \right].$$

$$\therefore E_y = Re\left[E_{y_0}'\left\{e^{j\left(\omega t + \frac{\pi x}{a} - \bar{\beta} z\right)} - e^{+j\left(\omega t - \frac{\pi x}{a} - \bar{\beta} z\right)}\right\}\right].$$

$$\therefore E_y = Re\left[E_{y_0}'\left[e^{j(\omega t - \bar{k}_1 \cdot \bar{r})} - e^{j(\omega t - \bar{k}_2 \cdot \bar{r})}\right]\right].$$

$$k_1 = -\frac{\pi}{a}\hat{a}_x + \bar{\beta}\hat{a}_z \quad , \quad \bar{k}_2 = \frac{\pi}{a}\hat{a}_x + \bar{\beta}\hat{a}_z.$$



→ Energy transfer taken place along the length of the waveguide.

→ $\bar{v}$

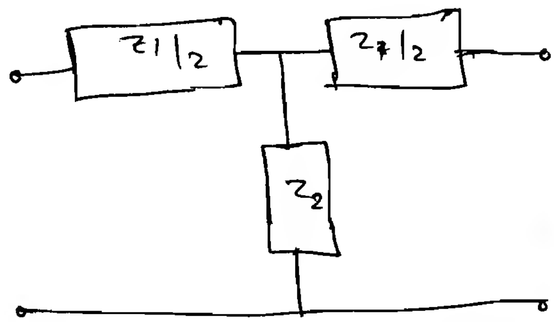
v_0

v_g ← group velocity. at which energy propogates.

$$\boxed{\bar{v} \cdot v_g = v_0^2}$$

☆ TWO WIRE TRANSMISSION LINE:

→ Even if the ports are interchange if the electric properties of the network or unaltered rather not disturb then a network is said to be symmetrical.



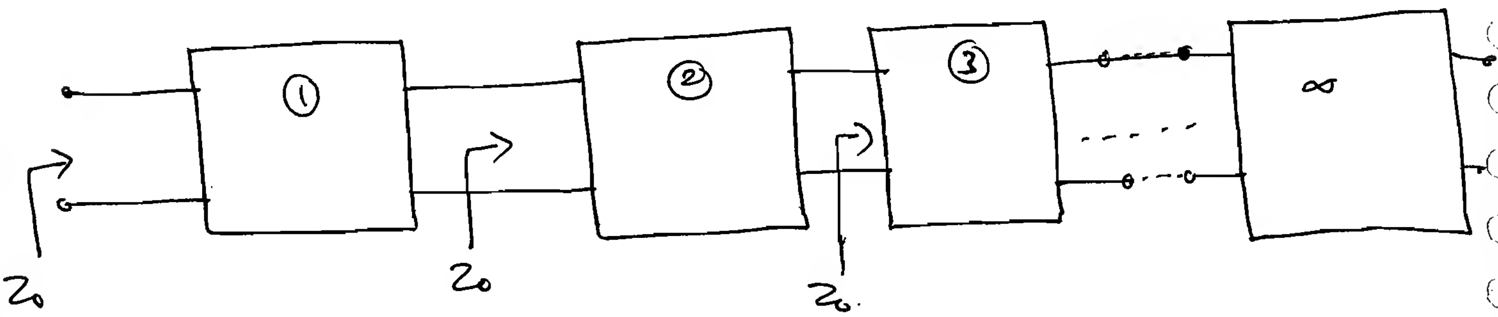
i/p ⟶ o/p

o/p ⟵ i/p

(Symm. N/W)

(Symm. N/W)

⇒ Defⁿ of Characterestic impedence Z_0:

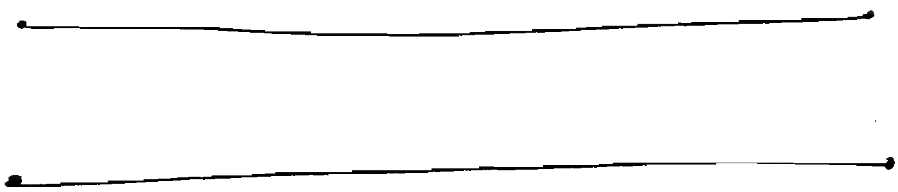


→ Infinite no. of identical symm. N/W are connected in cascade. The impedence seen at the i/p of the ① N/W is defined as Z_0.

* <u>Alternate Defn</u>:

→ When a symmetrical N/W is terminated by Z_0 then the impedence seen at the input of the network is also equals to Z_0. ~~These~~

→



i/p end
Sending end
Source end
Transmitting end

Output end
Receiving end
load end
terminating end.

→

 R Ω/m

 L → H/m.

 C → F/m.

Charges builts on wire.

distributed N/W.

 G ℧/m

→ The purpose of a transmission line is to transport energy from source to the load.

→ when a voltage is applied across this two wires the current passes through them when a current is passing through a conductor there exist voltage drop bet^n the conductor. significant of voltage drop indicate that the line is having series Resistance R.

→ when a current is passing through a conductor there exist magnetic field around the conductor. significant of magnetic field is indicates that line is having series inductance. L.

→ when a voltage is applied across this two wires the charges builts on the wires significant of this indicates that line is having shunt capacitance C.

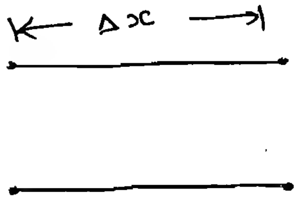
→ A capacitance never be an ideal one it has leakage conductance G.

→ R, L, C, G are not seen physically on the transmission line. they are distributed throughout the transmission line. therefore a transmission line is said to be an example for distributed network. and these are indicated

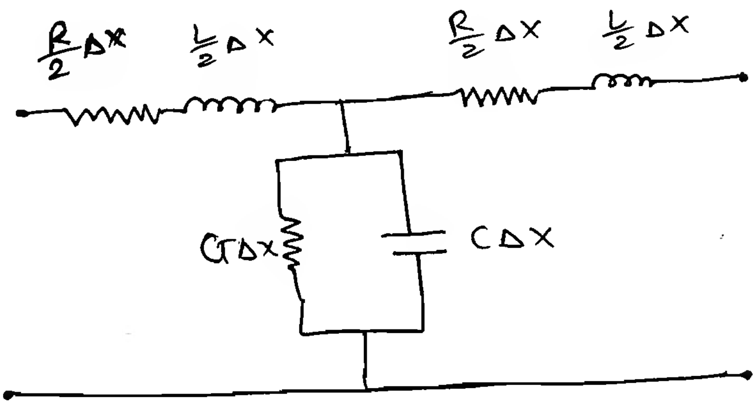
per unit length. If these are distributed uniformly then the transmission line is said to be uniform transmission line.

→ A section of a transmission line can be though an example for symitrical network.

→



Section of transmission line



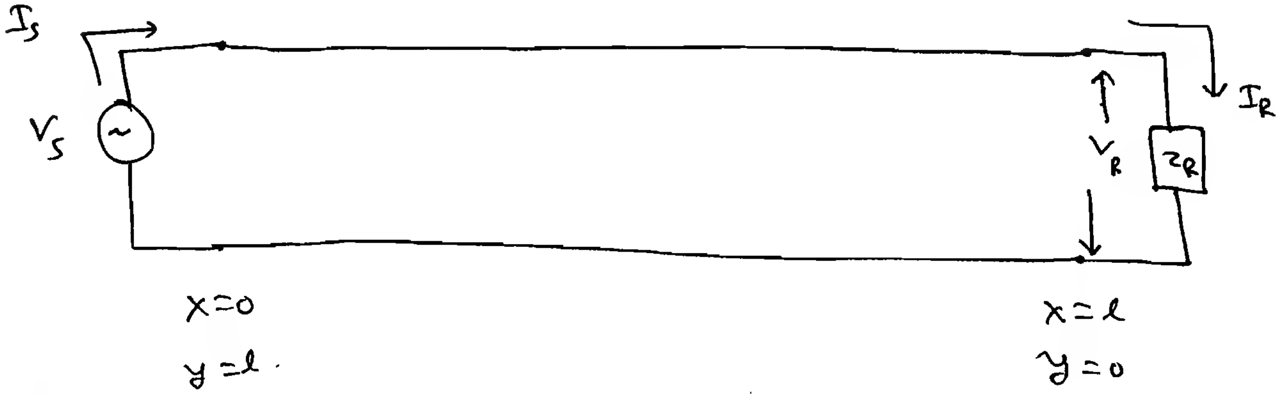
→ Symmetric T N/w representation.

→ Series Impedence $Z_s = (R + j\omega L)$ Ω/m.

→ ~~P~~ Shunt Impedence $Y = (G + j\omega C)$ ℧/m.

* Analysis of Transmission Line:

→ Transmission line analysis means finding out voltage and current at any point on the transmission line.



S: Sending

R: Received.

at $x=0$

$V=V_s$

$I=I_s$

x: distance measured from sending end.

y: distance measured from receiving end

at $x=l$

$V=V_R$

$I=I_R$

$$\boxed{\begin{array}{l} x=0 \text{ is corresponds to } y=l \\ y=0 \text{ is corresponds to } x=l. \end{array}}$$

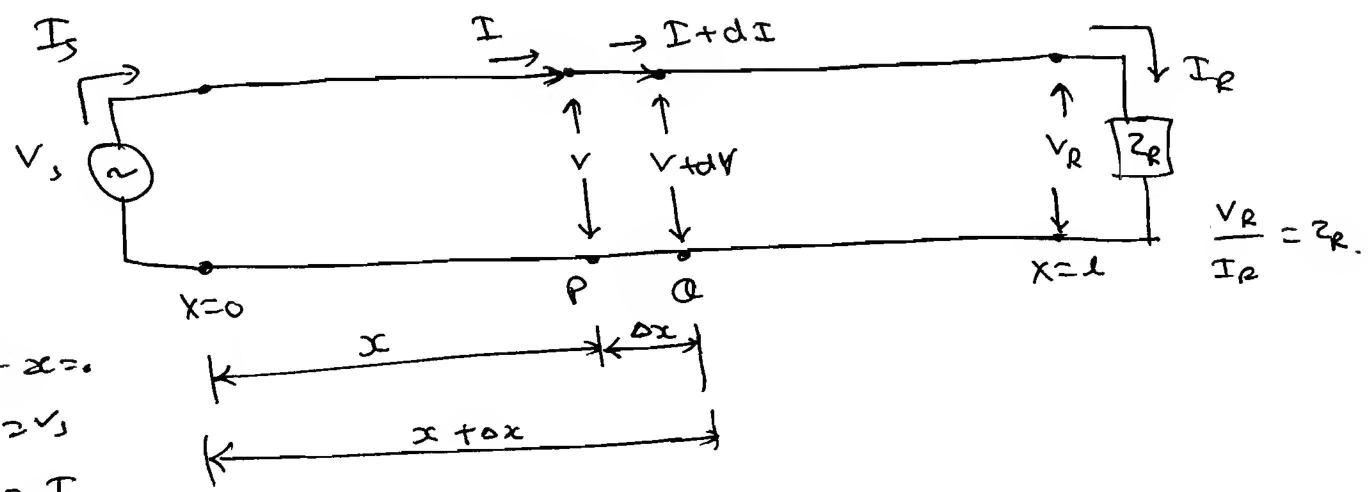
→ V: Voltage, I = current at any point on the line.

V, I ⇒ They can be represented in terms of V_s, I_s and x

They can be represented in terms of V_R, I_R, and y.

* **Expression for V & I at any point on the transmission line in terms of V_s, I_s & x:**

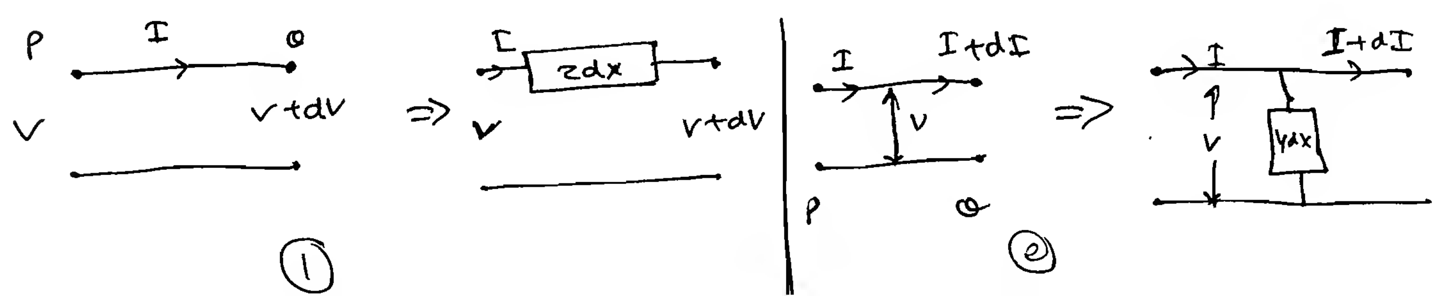
⟹



at x=0
$V = V_s$
$I = I_s$.

→ To calculate change in the voltage from P to Q we assume the current is constant over a small length dx and vice versa.

→ The change in the voltage from P to Q is because of the current I passing through the series impedence z.dx. Similarly change in the current from P to Q is because of the voltage applied across the shunt impedence Ydx.



→ KVL at ①

$V - (V + dV) = IZdx.$

$\therefore -dV = IZdx$

$\therefore \boxed{\frac{dV}{dx} = -IZ}$

KCL - ②

$I - (I + dI) = VYdx.$

$\therefore -dI = VYdx$

$\therefore \boxed{\frac{dI}{dx} = -VY.}$

differentiate again.

put

$\therefore \frac{d^2V}{dx^2} = -Z\frac{dI}{dx}$,

put

$\frac{d^2I}{dx^2} = -Y\frac{dV}{dx}.$

$\therefore -\cancel{\frac{d^2V}{dx^2}}\ \cancel{Z} + \cancel{Z}\cancel{d}$

$\therefore \boxed{\frac{d^2V}{dx^2} - \gamma^2 V = 0}$, $\boxed{\frac{d^2I}{dx^2} - \gamma^2 I = 0}$

→ $\gamma^2 = \sqrt{(R + j\omega L)(G + j\omega C)}.$

$\therefore$ ~~$\alpha = R + j\omega L$~~

~~$\beta = G + j\omega C.$~~

$\gamma^2 = \alpha + j\beta.$

γ: propagation const.

α = Attenuation const. (NP/m)

β = Phase shift const. (rad/m).

→ The above eq^n's in the form of harmonic eq^n. The sol^n of a harmonic eq^n may take either sine (or) cosine (or) exponent.

$\therefore \frac{d^2V}{dx^2} - \gamma^2 V = 0$, $\frac{d^2I}{dx^2} - \gamma^2 I = 0$

$m = \pm\gamma$ $\qquad m = \pm\gamma$

$$\therefore \boxed{V = C_1 e^{-\gamma x} + C_2 e^{+\gamma x}}$$

$$\frac{\gamma}{z} = \frac{\sqrt{zy}}{z} = \sqrt{\frac{zy}{z^2}}$$

$$\frac{\gamma}{z} = \sqrt{\frac{y}{z}} = \frac{1}{z_0}$$

$$\boxed{z_0 = \sqrt{\frac{z}{y}}}$$

$$\therefore I = C_3 e^{-\gamma x} + C_4 e^{+\gamma x}.$$

$\therefore$ from ① $\quad I = -\frac{1}{z}\frac{dV}{dx}.$

$$\therefore I = -\frac{1}{z}\left[-\gamma C_1 e^{-\gamma x} + \gamma C_2 e^{+\gamma x}\right]$$

$$\therefore I = \gamma/z \left[C_1 e^{-\gamma x} - C_2 e^{+\gamma x}\right].$$

$$\therefore \boxed{I = \frac{1}{z_0}\left[C_1 e^{-\gamma x} - C_2 e^{+\gamma x}\right]}$$

→ C_1 & C_2 are evaluated by using the conditions (i.e.) at $x=0$ $V=V_s$ & $I=I_s$.

by simplifying we get.

$$\therefore V = V_s \cosh \gamma x - I_s z_0 \sinh \gamma x$$

$$I = I_s \cosh \gamma x - \frac{V_s}{z_0} \sinh \gamma x.$$

$$\therefore \boxed{\begin{aligned} V &= C_1 e^{-\gamma x} + C_2 e^{\gamma x} \\ I &= \frac{1}{z_0}\left[C_1 e^{-\gamma x} - C_2 e^{\gamma x}\right] \end{aligned}}$$

* Infinite line and the defination of Characteristic Impedence.

→ Infinite line means line length is ∞.

→ An infinite line can be thought of infinite no of small section of subsection connected in cascade.

→ We know that each subsection is equal to a symmetrical N/W. Therefore an infinite line can be thought of infinite no. of identical symmetrical N/W. which are connected in cascade. Therefore, we define input impedence of an infinite line is equals to characteristic impedence.

→ $V_{(at\ x=l)} = V_R = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l.$

$I_{(at\ x=l)} = I_R = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l.$

$$\therefore \frac{V_R}{I_R} = Z_0 = \frac{V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l}{I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l}.$$

∴ Cross multiply

we get $\frac{V_s}{I_s} = Z_0.$

$$\therefore Z_0 I_s \cosh \gamma l - V_s \sinh \gamma l = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l.$$

$$\therefore Z_0 I_s \cancel{[\cosh \gamma l + \sinh \gamma l]} = V_s \cancel{[\cosh \gamma l + \sinh \gamma l]}$$

$$\therefore \boxed{\frac{V_s}{I_s} = Z_0.}$$

→ An infinite line is equal to a finite line when the finite line is terminated by Z_0.

→ i.e. when a finite line transmission line is terminated by Z_0 then the impedence seen at the input of the transmission line is also equals to Z_0

→ when $Z_R = Z_0 \Rightarrow V_s = I_s Z_0$.

$V = V_s \cosh \gamma x - V_s \sinh \gamma x$..

$\therefore$ $\cancel{V_s}$ $\cancel{V_0}$ very

$$\boxed{\begin{aligned} V &= V_s e^{-\gamma x} \\ I &= I_s e^{-\gamma x}. \end{aligned}}$$

These are valid when $Z_R = Z_0$. only.

→ If 'l' is called physical length of the line, the 'βl' is called 'electrical length'.

$l \to m$

$\beta \to$ rad/m.

$\beta l \to \frac{rad}{\cancel{m}} \cdot \cancel{m} =$ rad (or) degree.

Ex-1: The physical length of the transmission line is $\lambda/8$. find the electrical length.

Ans: $l: \lambda/8$

$\therefore$ $\beta l = \frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{8\,4} = \frac{\pi}{4}$.

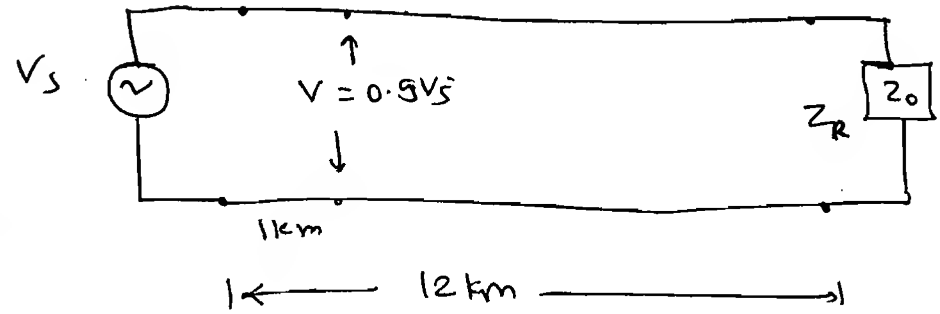
$\therefore$ $\boxed{\text{electrical length} = \frac{\pi}{4}.}$

Ex 2

→ A 12 km long line is terminated by Z_0. The Voltage at 1 km from the sending end is 10% below than at the sending end. Find voltage across the load impedence in terms of the % of the sending end voltage.

Ans: 28%.

Ans:



10% below
$= V_s - 0.1V_s$
$= 0.9V_s.$

→ at 1 km $V = 0.9\, V_s.$

∴ the line is terminated in Z_0.

∴ $V = V_s e^{-\gamma x}$

at $x = 1\text{km} \Rightarrow V = 0.9 V_s = V_s e^{-\gamma(1)}$

∴ $\boxed{0.9 = e^{-\gamma}}$

at $x = 12\text{km}$ $V = V_s e^{-\gamma(12)}$

∴ $V = V_s (0.9)^{12} = 0.28\, V_s$

∴ $\boxed{V_{(x=12km)} = 28\%\ \text{of}\ V_s}$

$\left.\begin{matrix}\gamma \\ Z_0\end{matrix}\right\} \rightarrow$ Secondary Constant

$R, G, L \rightarrow$ Primary Constant.

* Distortion:

→ The Purpose of a transmission line is to transport band of freq.

→ α is a f^n of of freq. therefore different freq. Components undergoes different attenuation levels. This is said to be freq. Distortion.

→ Undergoing different freq. Components with the different phase velocity is called delay distortion sin $V_p = \omega/\beta$.

$$\text{Distortion} \begin{cases} \text{Frequency} \\ \text{Delay } v_p = \omega/\beta \end{cases}$$

$v_p = \frac{2\pi f}{\beta}$.

$\alpha = \text{real}(\gamma)$

$= \text{Real}\sqrt{(R+j\omega L)(G+j\omega C)}$

$= f(R, L, G, C, \omega)$.

→ when the line is free from delay distortion and freq distortion then it is said to be distortion less transmission line.

* **Distortionless condition:**

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$\therefore$ $\gamma = \sqrt{L\left[\frac{R}{L} + j\omega\right] \cdot C\left[\frac{G}{C} + j\omega\right]}$

if we choose $\frac{R}{L} = \frac{G}{C}$.

$\therefore$ $\gamma = \sqrt{LC\left[\frac{R}{L} + j\omega\right]^2}$ or $\gamma = \sqrt{LC\left[\frac{G}{C} + j\omega\right]^2}$

$\therefore$ $\gamma = \sqrt{LC}\left[\frac{R}{L} + j\omega\right]$. (or) $\gamma = \sqrt{LC}\left[\frac{G}{C} + j\omega\right]$.

$\therefore$ $\gamma = R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$ or $\gamma = G\sqrt{\frac{L}{C}} + j\omega\sqrt{LC}$.

$\therefore$ $\boxed{\alpha = R\sqrt{\frac{C}{L}}}$ (or) $\boxed{G\sqrt{\frac{L}{C}}}$ Np/m.

$\therefore$ $\boxed{\beta = \omega\sqrt{LC}}$ m/s. $\Rightarrow$ $V_p = \frac{1}{\sqrt{LC}}$

→ α and v_p are independent of frequency.

$\therefore$ $\boxed{\frac{R}{L} = \frac{G}{C} \;:\; \text{Distortionless condition.}}$

⇓

The same condition is also be valid for min. attenuation (or) Low loss.

$\therefore$ $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L\left(\frac{R}{L} + j\omega\right)}{C\left(\frac{G}{C} + j\omega\right)}}$

$\therefore$ $\boxed{Z_0 = \sqrt{\frac{L}{C}}\ \Omega}$

→ Delay in the transmission line is the geometric mean of L & C. (or)

$\therefore \boxed{\text{Delay} = \sqrt{LC} \text{ s/m.}}$

$\beta = \omega\sqrt{LC}$

$\therefore v_p = \omega/\beta.$

Ex-1 A distortionless transmission line has an attenuation constant of 20m Np/m. The phase velocity on the line is 0.6 of the velocity of light. assume characteristic impedence of the line is 50 Ω. Find primary constants. if

Ans: $\alpha = 20\text{m Np/m.}$

$\alpha = 20 \times 10^{-3}$ Np/m.

$\therefore v_0 = \omega/\beta = 0.6 \times 3 \times 10^8 = 1.8 \times 10^8.$

$\therefore \frac{\omega}{\beta} = 1.8 \times 10^8$ m/s.

$\therefore Z_0 = 50\ \Omega. = \sqrt{\frac{L}{C}}.$

$\therefore \alpha = R\sqrt{\frac{C}{L}}$, (or) $\alpha = G\sqrt{\frac{L}{C}}.$

$\therefore \cancel{\alpha = G Z_0}$

$\therefore \cancel{G = \frac{20 \times 10^{-3}}{50}}$

$\cancel{\boxed{G = 0.4 \times 10^{-3}}}$ Ʊ/km

$\therefore \quad G\,R = \alpha / Z_0 \quad \Omega/m.$

$\therefore \quad G\,R = \dfrac{20 \times 10^{-3}}{50} = \cancel{0.6 \times 10^{-6}}$

$\therefore \quad \boxed{G = 0.4 \times 10^{-3} \ \text{℧}/m.}$

$\therefore \quad R = \alpha . Z_0.$

$\therefore \quad R = 20 \times 10^{-3} \times 50 = 1000 \times 10^{-3}$

$\therefore \quad \boxed{R = 1 \ \Omega/m}$

$\therefore \quad L = \dfrac{Z_0}{v_p}$

$\therefore \quad L = \dfrac{50 \times 1}{0.6 \times 10^{8} \times 3} = \dfrac{50 \times 10^{-8}}{1.8}.$

$\therefore \quad \boxed{L = 27.6 \times 10^{-9} \ H/m.}$

$\therefore \quad C = \dfrac{1}{Z_0 v_p}. \quad F/m.$

$\therefore \quad C = \dfrac{1}{50 \times 3 \times 0.5 \times 10^{+8}}$

$C = \dfrac{100}{75} \times 10^{-8} \times 10^{-2}$

$\therefore \quad C = 1.3 \times 10^{-10} \ F.$

$\boxed{C = 133.33 \ pF}$

* For a lossless line $\boxed{\alpha = 0.}$

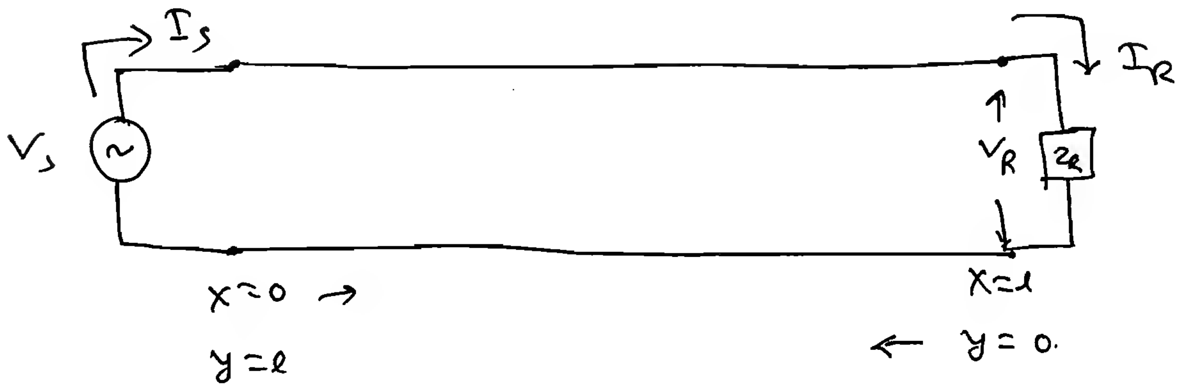
$\gamma = j\beta$

$\sinh \gamma l = \sinh (j\beta l)$

$= j \sinh (\beta l).$

$\cosh \gamma l = \cosh (j\beta l) = \cos \beta l.$

* Expression for V & I at any point on the transmission line interms of V_R, I_R, y.



at $y \Rightarrow 0$, $V = V_R$
$I = I_R$.

$\therefore V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x.$

$I = I_s \cosh \gamma x - \dfrac{V_s}{Z_0} \sinh \gamma x.$

$\Downarrow$

Replace

$x \rightarrow -y$

$V_s \rightarrow V_R.$

$I_s \rightarrow I_R.$

$\therefore V = V_R \cosh \gamma y + I_R Z_0 \sinh \gamma y.$

$\therefore I = I_R \cosh \gamma y + \dfrac{V_R}{Z_0} \sinh \gamma y.$

at $y = l \Rightarrow \boxed{V = V_s, \quad I = I_s.}$

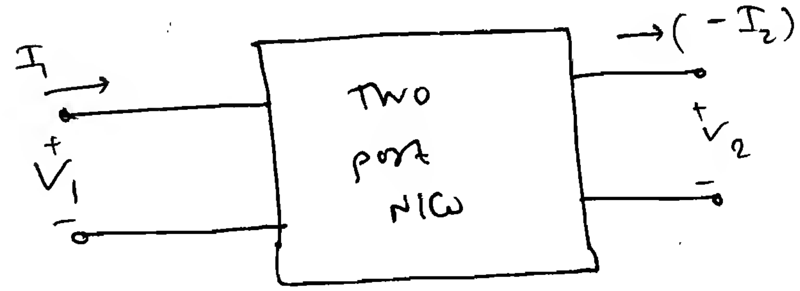
input end

$$\therefore \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \overset{A}{\boxed{\cosh \gamma l}} & \overset{B}{\boxed{Z_0 \sinh \gamma l}} \\ \underset{C}{\boxed{\frac{1}{Z_0} \sinh \gamma l}} & \underset{D}{\boxed{\cosh \gamma l}} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}.$$

→ For a lossless line $\alpha = 0 \Rightarrow \gamma = j\beta.$

$$\underset{I_1}{\overset{V_1}{\begin{bmatrix} V_s \\ I_s \end{bmatrix}}} = \begin{bmatrix} \overset{A}{\boxed{\cos \beta l}} & \overset{B}{\boxed{j Z_0 \sin \beta l}} \\ \underset{C}{\boxed{\frac{j}{Z_0} \sin \beta l}} & \underset{D}{\boxed{\cos \beta l}} \end{bmatrix} \underset{-I_2}{\overset{V_2}{\begin{bmatrix} V_R \\ I_R \end{bmatrix}}}.$$

→



$$V_1 = A V_2 - B I_2.$$

$$I_1 = C V_2 - D I_2.$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

→ $A = \left. \frac{V_1}{V_2} \right|_{I_2 = 0}$ → $B = \left. -\frac{V_1}{I_2} \right|_{V_2 = 0}$

→ $C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0}$ → $D = \left. \frac{-I_1}{I_2} \right|_{V_2 = 0}.$

→ As shown above a section of a transmission line is satisfying reciprocity condition, ~~of~~ symmetry condition of ABCD parameters. we can also prove that a section of a transmission line is an example for symmetrical and reciprocal

Ex-2: For a loss-less transmission line of a length $\lambda/8$. Find ABCD parameters.

Ans: Assume characteristic impedence of the in $50\,\Omega$.

→ as loss-less $\Downarrow$ $\alpha = 0$ $\quad \gamma = \alpha + j\beta$

$\therefore \Rightarrow \gamma = j\beta$.

$\therefore$ $\underline{Z_0 = 50\,\Omega}$

For a loss-less line $\alpha = 0 \Rightarrow \gamma = j\beta$.

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cos \frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{8} & j\,50 \sin \frac{2\pi}{\lambda} \times \lambda/8 \\ \frac{j}{50} \sin \frac{2\pi}{\cancel{\lambda}} \times \cancel{\lambda}/\cancel{8}_4 & \cos \frac{2\pi}{\lambda} \times \lambda/\cancel{8}_4 \end{bmatrix} \begin{bmatrix} V_R \\ -I_R \end{bmatrix}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & j\frac{50}{\sqrt{2}} \\ \frac{j}{50\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} V_R \\ -I_R \end{bmatrix}$$

☆ Impedence:

→ Impedence at any point on the transmission line looking towards the load is the ratio of voltage to the current at that point Z_y is the Impedence at a distance y from a load end.

$$Z_y = \frac{V_R \cosh \gamma y + I_R Z_0 \sinh \gamma y}{I_R \cosh \gamma y + \frac{V_R}{Z_0} \sinh \gamma y}.$$

→ Divide the numarator and denominator by $I_R \cosh \gamma y$. . and use $\frac{V_R}{I_R} = Z_R$ and simpiibying.

$$\boxed{Z_y = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma y}{Z_0 + Z_R \tanh \gamma y} \right]}$$

→ For a loss less line $\alpha = 0$.

$$\therefore \boxed{Z_y = Z_0 \cdot \frac{Z_R + Z_0 \tanh \beta \ell y}{Z_0 + Z_R \tanh \beta y}}$$

→ When $y = l \Rightarrow$ input and Z_y is denoted as Z_{in}

(or) $\boxed{Z_{in} = \frac{V_s}{I_s}}$.

→ For a lossy ~~gress~~ line,

$$\therefore \boxed{Z_{in} = Z_0 \cdot \frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}}.$$

→ For a loss-less line,

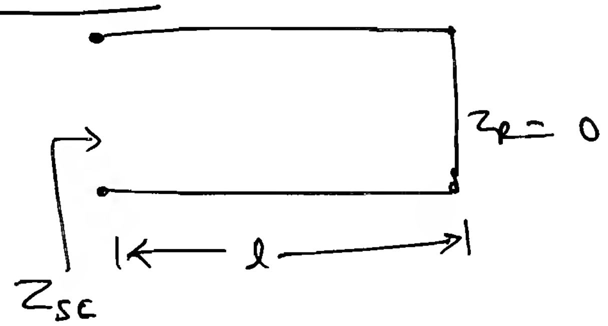
$$\therefore \boxed{Z_{in} = Z_0 \cdot \frac{Z_R + j Z_0 \tan \beta l}{Z_0 + j Z_R \tan \beta l}}.$$

→ When a transmission line is terminated by Z_0, then, the impedence seen at any point on the transmission line and also the input impedence is same as Z_0 weither the line is lossy (or) lossless.

→ When a finite length transmission line is open circuited at the terminating end is called OC line then the impedence seen at input of the transmission line is designted by Z_{oc}.

→ When a finite length Tx line is Short Ckted at the terminating end is called as SC line then the impedence seen at the input of the transmission line is designated by Z_{sc}.

→ ① SC line

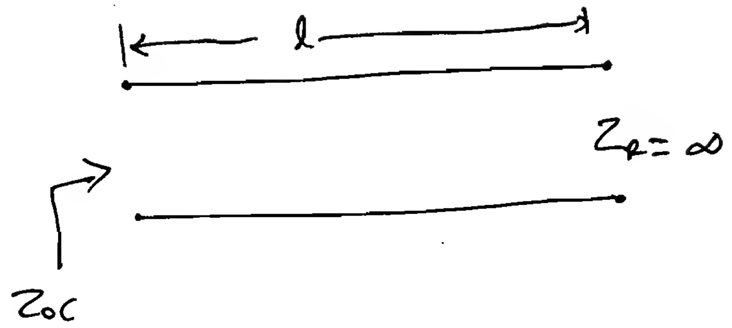


$$\therefore \quad Z_{sc} = Z_0 \tanh \gamma l \ \Omega$$

For a loss less line.

$$Z_{sc} = j Z_0 \tan \beta l \ \Omega.$$

② OC line.



$$Z_{oc} = Z_0 \coth \gamma l \ \Omega.$$

For a loss less line.

$$Z_{oc} = -jZ_0 \cancel{\tan}\ \cot \beta l. \ \Omega.$$

$$Z_{oc} = -jZ_0 \cot \beta l \ \Omega.$$

→ Whether the line is lossy (or) lossless,

$$\boxed{Z_{oc} \cdot Z_{sc} = Z_0^2}$$

→ Z_0: Char. impedance 50 Ω line, 75 Ω line etc.

βl : Electrical length.

→ $\left.\begin{array}{l}\tan \beta l \\ \cot \beta l\end{array}\right\}$ They assume all possible values ranging from $-\infty$ to ∞, depending upon the value βl.

→ For a loss less line.

$$Z_{sc} = j Z_0 \tan \beta l \ \Omega$$

$$Z_{oc} = -j Z_0 \cot \beta l \ \Omega.$$

Rectance.

→ $Y_{sc} = \boxed{-\frac{j}{Z_0}\cot \beta l}$ ℧

$Y_{oc} = \boxed{\frac{j}{Z_0}\tan \beta l}$ ℧

Suseptance.

→ **Admitance** | **Impedences**

$\frac{1}{j\omega L} = -j\boxed{B_L}$ ℧

$j\omega L \ \Omega = jX_L \ \Omega$

$j\omega C = +j\boxed{B_C}$ ℧

$\frac{1}{j\omega C} \ \Omega = -jX_C \ \Omega.$

Suspetance

→ We conclude that a section of a lossless transmission line either it is open circuited (or) SC can act as circuit reactive eliment (or) circuit suseptive element. Desire Reactance (or) Susptance can be achieve by propery choosing length of the transmission line this sections are named as stubs. and are used in the impedence matching techniques hence the name stub matching.

Ex-1 A loss less $\lambda/8$ transmission line is SC. What type of reactance the line indicates. at 80 KHz find the value of equivalent passive component.
assume $Z_0 = 50\ \Omega$.

Ans: as S.C.

$$Z_{sc} = jZ_0 \tan\beta l \ \Omega.$$

$\therefore$ $l = \lambda/8$, $\beta = \frac{2\pi}{\lambda}$.

$\therefore$ $Z_{sc} = j\ 50 \tan\left(\frac{\pi}{4}\right)$.

$$Z_{sc} = j50\Omega$$

$\therefore$ line offers inductive reactance.

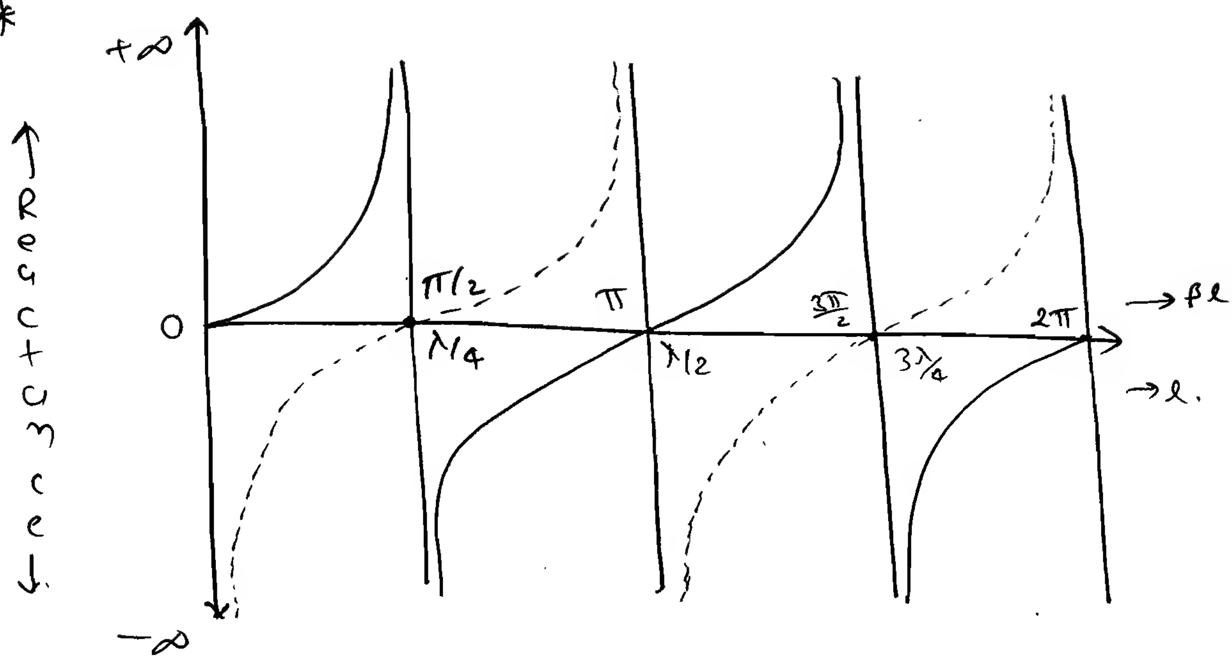
$$j50 = j\omega L$$

$$2\pi f L = 50$$

$\therefore$ $L = \frac{\cancel{50}}{2 \times \pi \times \cancel{80} \times 10^3}$

$\therefore$ $\boxed{L = 0.159\,mH}$

*



——— Variation of Z_{sc}.
------ Variation of Z_{oc}.

$Z_{sc} = jZ_0 \tan\beta l$.

$\therefore Z_{oc} = -jZ_0 \cot\beta l$.

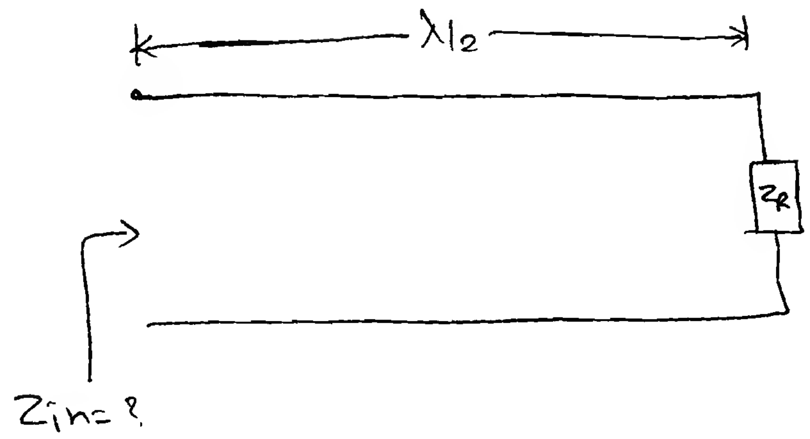
→ figure shows the variation of Zoc and Zsc as a function of physical length (or) electrical length. when the line length is varied from 0 to λ/2. Zoc and Zsc assumes all possible reactances ranging from −∞ to +∞ that desired reactance can be achieve by properly choosing length of the transmission line.

→ when the line length is varied from 0 to λ/4 sc line would offer inductive reactance and oc line would offer capacitive reactance.

EMTL

* Properties of λ/2 and λ/4 lines.

→ Consider a lossless line



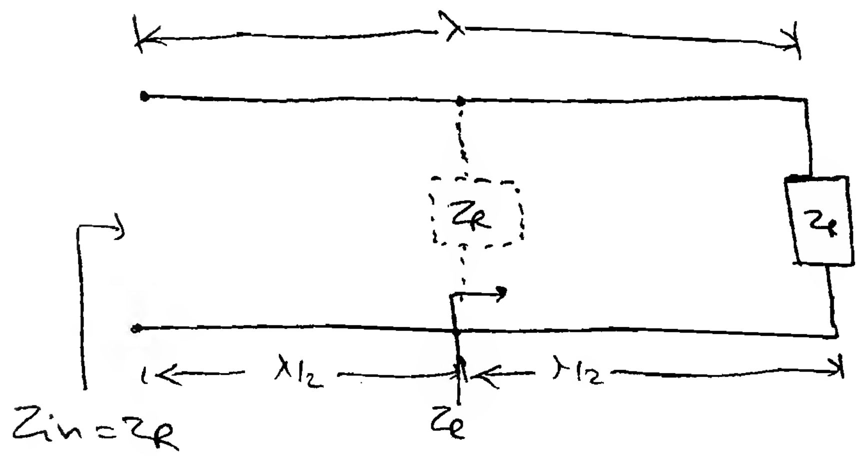
→ $l = \lambda/2$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$\tan\beta l = \tan\pi = 0.$$

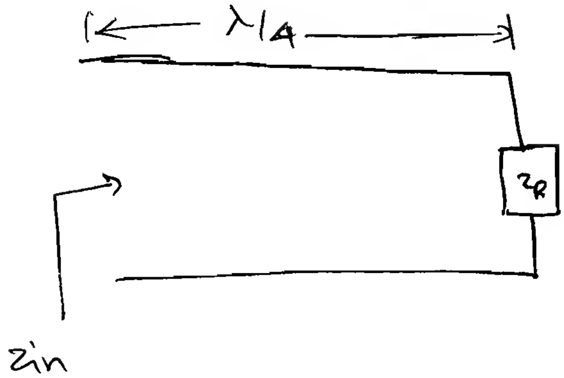
$\therefore$ $\boxed{Z_{in} = Z_R.}$ $\qquad \left(\because Z_{in} = Z_0 \dfrac{Z_R + jZ_0\tan\beta l}{Z_0 + jZ_R\tan\beta l}\right).$

→



→ The input impedence of a $\frac{n\lambda}{2}$ line is same as λ/2 line and is equals to load impedence.

⇒ $\lambda/4$ line.



$$l = \lambda/4.$$

$$\therefore \beta l = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}.$$

$$\tan\beta l = \tan\frac{\pi}{2} = \infty.$$

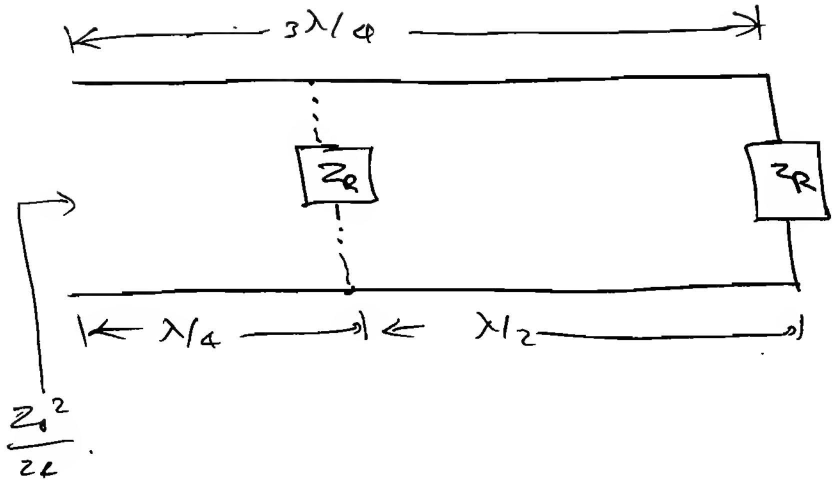
$$\therefore Z_{in} = Z_0 \cdot \frac{\frac{Z_R}{\tan\beta l} + jZ_0}{\frac{Z_0}{\tan\beta l} + jZ_R}.$$

$$\therefore Z_{in} = \frac{Z_0^2}{Z_R}.$$

when $Z_R =$	$Z_{in} = ?$
$0\,\Omega$ (SC)	$\infty\,\Omega$ (OC)
$\infty\,\Omega$ (OC)	$0\,\Omega$ (S.C).

→ A quarter wave line is also called as Impedence transformer. because it transforms High impedence to low impedence and vice versa.

→

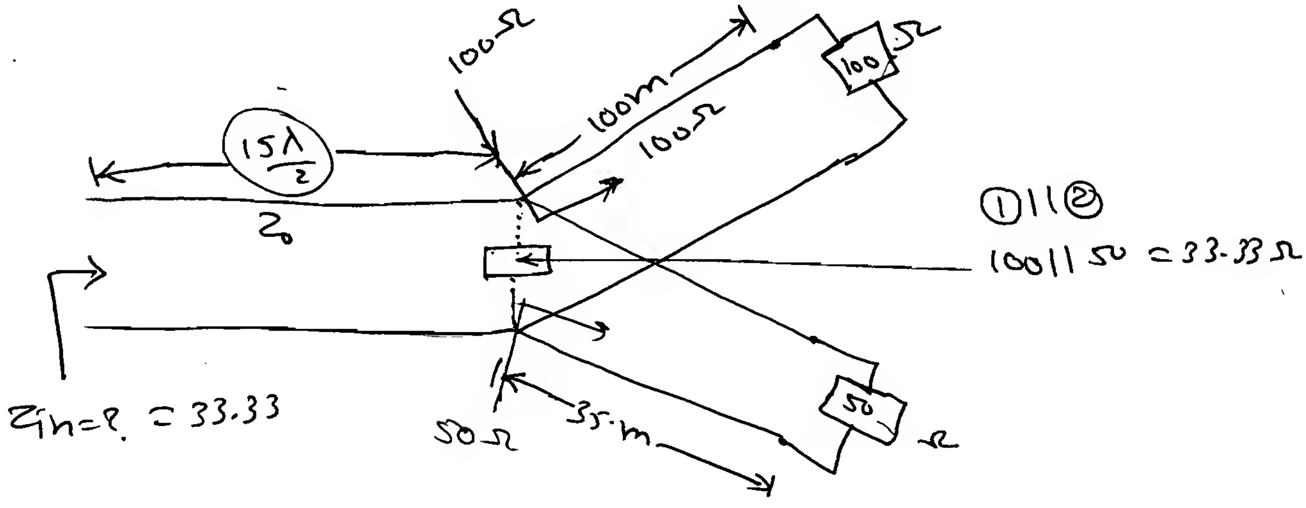


→ The input impedence of a $\boxed{\frac{n\lambda}{4}}$ line is same as λ/4 line and is equals to $\boxed{\frac{Z_0^2}{Z_R}}$. where $\boxed{n \text{ is odd.}}$

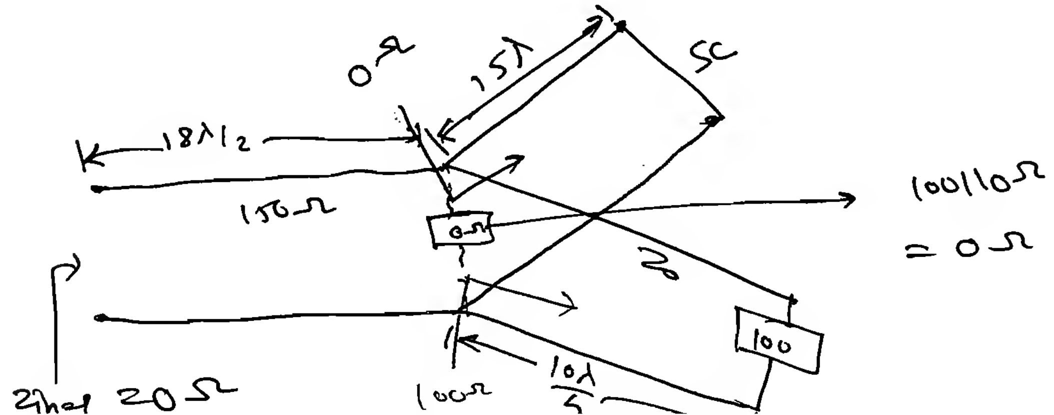
Ex-1: Find the input impedence of the following transmission line. Assume lossless line.

→

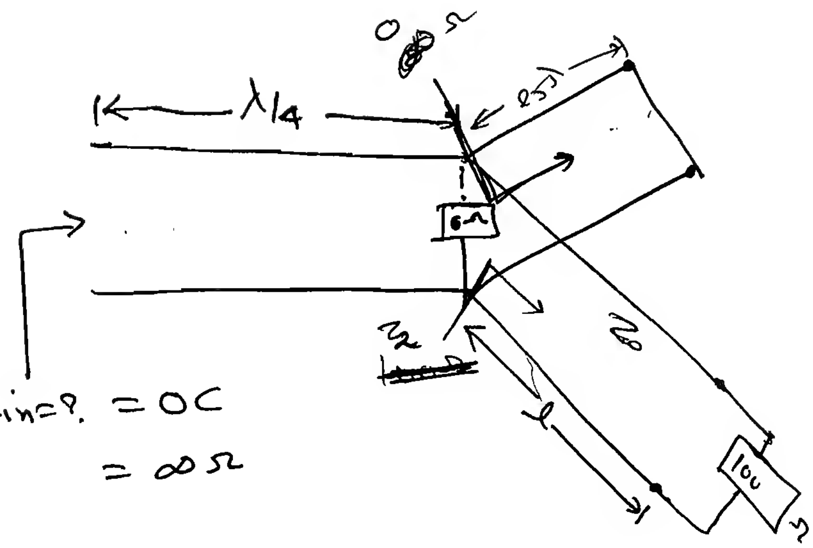
①



②



③



$Z_{in} = ?\ = OC$

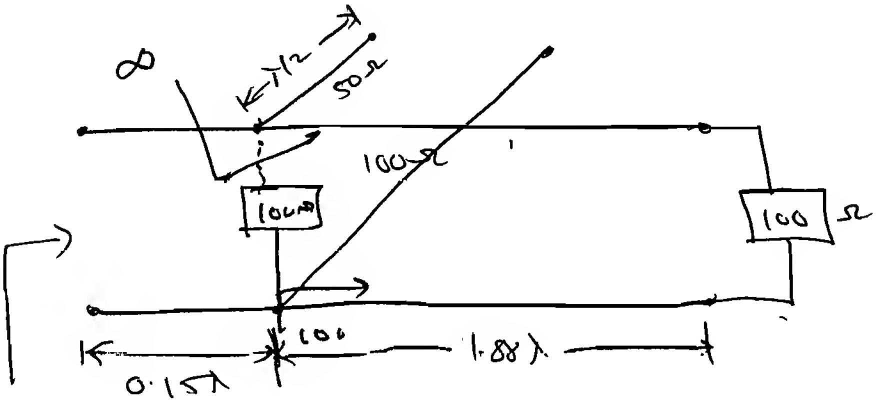
$= \infty\,\Omega$

$\infty \parallel 100$

$= 0\,\Omega$

$\boxed{Z_2 \parallel 0\,\Omega = 0\,\Omega}$

④



$Z_{in} = 100\,\Omega$

$\infty \parallel 100 = 100.$

$100\,\Omega$ O-C $= 100\,\Omega$

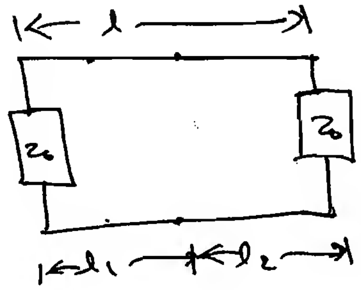
⑤ The two ends of a lossless trans. line is terminating Z_0. where Z_0 in the characteristic impedence of the line. Find the impedence at the middle point and at any point on the line.

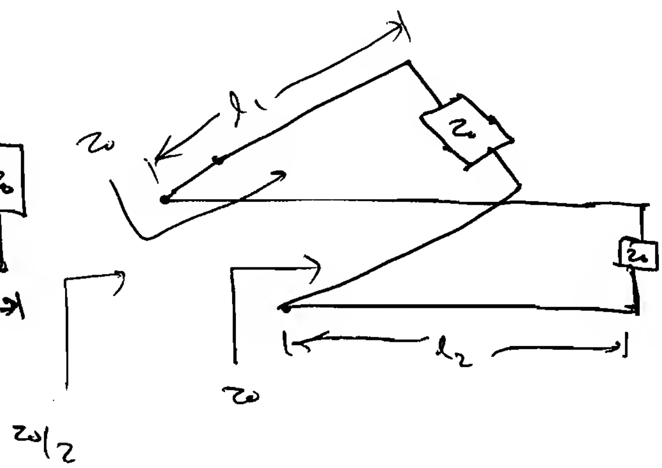
(a) Z_0, Z_0

(b) $\frac{Z_0}{2}, Z_0.$

(c) $Z_0, Z_0/2.$

(d) $\frac{Z_0}{2}, \frac{Z_0}{2}.$





* Reflection

→ Z_y is the impedence at any point on the transmission line looking towards the load and is given by

$$Z_y = Z_0 \frac{Z_l + Z_0 \tanh \beta l}{Z_0 + Z_l \tanh \beta l}.$$

→ When $Z_l = Z_0 \Rightarrow Z_y = Z_0$

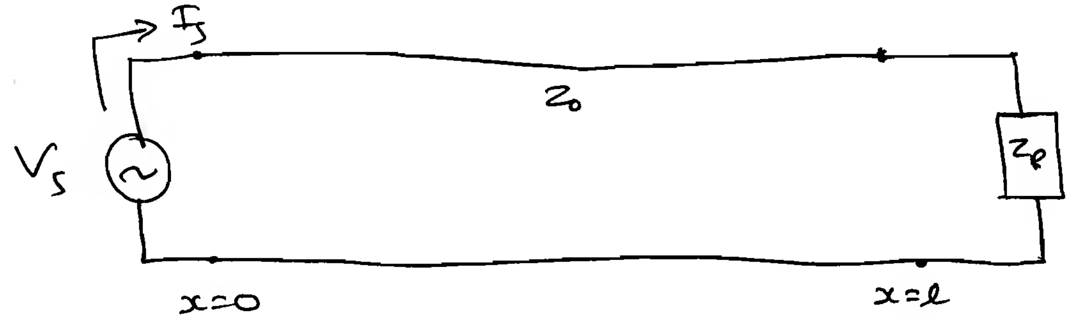
⇒ Impedence is Matched (or)
Uniform (or)
Regular (or)
Continuous.

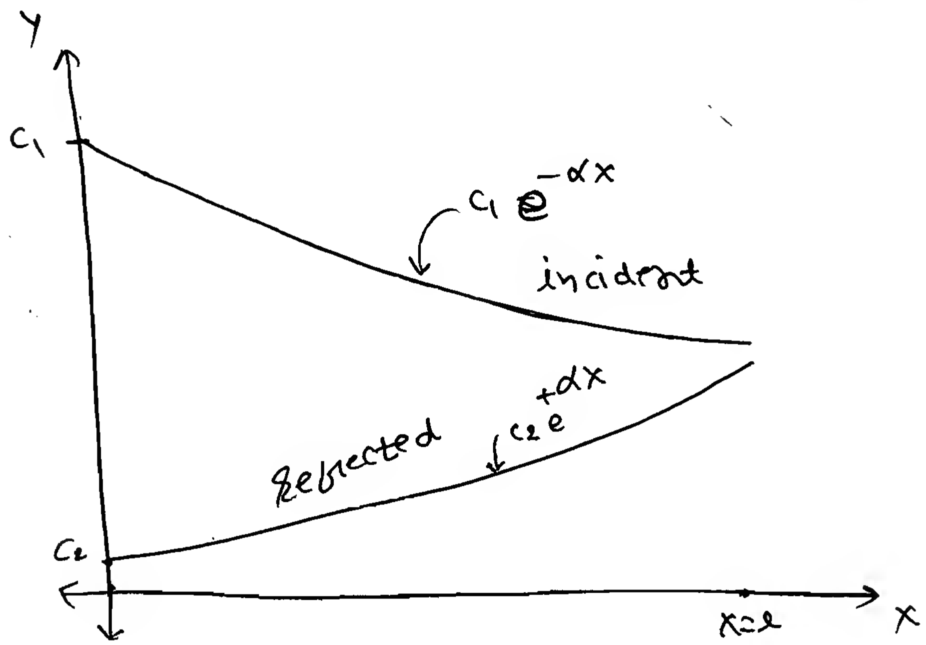
→ When $Z_l \neq Z_0$

⇒ Z_y Changes its value from point to point on the line.

⇒ Impedence is mismatched (or)
discontinuous (or)
Irregular (or)
Non-uniform.

→ Whereever there exist impedence discontinuity (or) irregularity (or) mismatched (or) nonuniformility there exist reflection. this reflection will travel from load to the source





→ The Voltage at anypoint on the line is given by

$$V = C_1 e^{-\gamma x} + C_2 e^{+\gamma x}$$

$$\therefore V = \underbrace{C_1 e^{-\alpha x} \cdot e^{-j\beta x}}_{\substack{\text{Term - 1} \\ \text{incident}}} + \underbrace{C_2 e^{\alpha x} \cdot e^{j\beta x}}_{\substack{\text{Term 2} \\ \text{Reflected.}}}$$

→ Term - 1

$$\left| C_1 e^{-\alpha x} \cdot e^{-j\beta x} \right| = C_1 e^{-\alpha x}$$

→ Term - 2

$$\left| C_2 e^{\alpha x} \cdot e^{j\beta x} \right| = C_2 e^{\alpha x}.$$

when $Z_R = Z_0$.

$V = V_s . e^{-\gamma x}$ + No second term (reflection is zeros.

→ Term-① in its mathematical form indicates an incident wave which is propogating from source to the load while it is propagating its amplitude is decreasing exponetially.

→ The Term-② in its mathematical form indicates reflected wave which is propogating from load to the source while it is propogating its ampitude is decreasing exponetially.

→ when $Z_R = Z_0$ Reflections are zero. i.e. there exist only one wave i.e. incident wave.

→ $V = C_1 e^{-\gamma x} + C_2 e^{+\gamma x}$

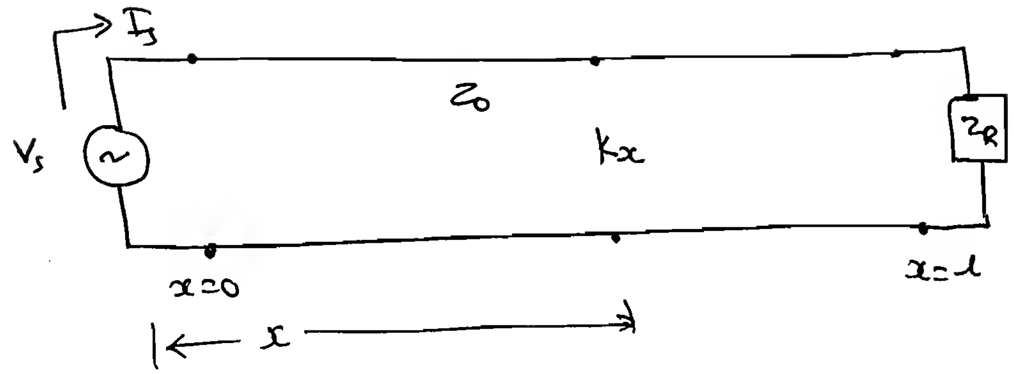
$$\therefore V(x,t) = Re\left[\underbrace{C_1 e^{-\alpha x} . e^{j(\omega t - \beta x)}}_{\text{incident wave } (+x)} + \underbrace{C_2 e^{\alpha x} . e^{j(\omega t + \beta x)}}_{\text{Reflected wave } (-x)} \right].$$

→ Reflection coefficient:

→ It is a ratio of reflected wave voltage to the incident wave voltage. (K_x).

→ K_x is the reflection coefficient is define in a distance x from the sending end.

→

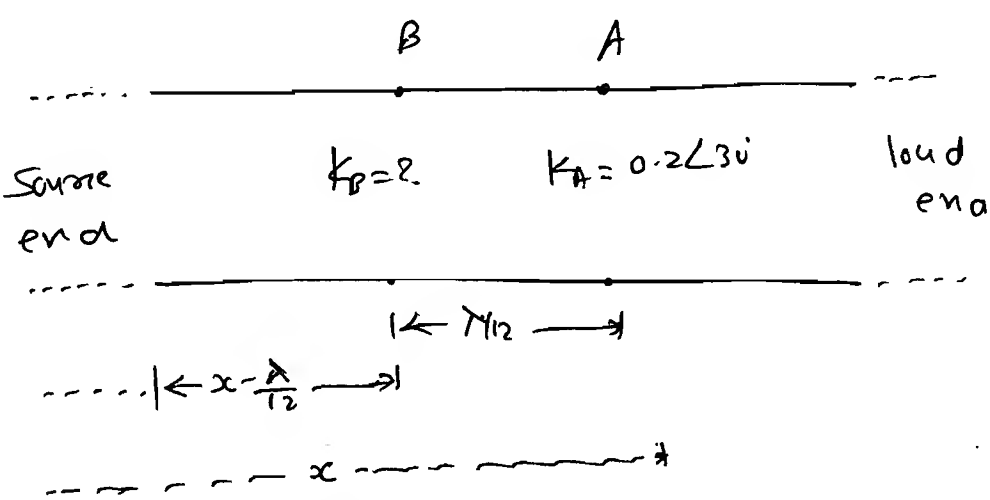


$$\rightarrow K_x = \frac{C_2 e^{\gamma x}}{C_1 e^{-\gamma x}} = \frac{C_2}{C_1} e^{2\gamma x} = \frac{C_2}{C_1} e^{2\alpha x} \cdot e^{2j\beta x}.$$

For, a loss less line $\boxed{K_x = \frac{C_2}{C_1} \cdot e^{2j\beta x}}$.

Ex-1 A finite length transmission line is lossless and terminated by an unknown load impedence. Reflection coefficient measured at point A is given by $0.2\angle 30°$. What is it value at a distance of $\frac{\lambda}{12}$ from the above point towards the sending end.

Ans:



$$K_A = \frac{C_2}{C_1} \cdot e^{2j\beta x}.$$

$$K_B = \frac{c_2}{c_1} \cdot e^{2j\beta\left(x - \frac{\lambda}{12}\right)} = \frac{c_2}{c_1} e^{2j\beta x} \cdot e^{-j2\left(\frac{2\pi}{\lambda}\right)\cdot\left(\frac{\lambda}{12}\right)}$$

$$= \frac{c_2}{c_1} \cdot e^{2j\beta x} \cdot e^{-j\pi/3}.$$

$$\therefore \frac{k_B}{k_A} = \frac{\cancel{\frac{c_2}{c_1}} \cdot \cancel{e^{2j\beta x}} \cdot e^{-j\pi/3}}{\cancel{\frac{c_2}{c_1}} \cdot \cancel{e^{2j\beta x}}}$$

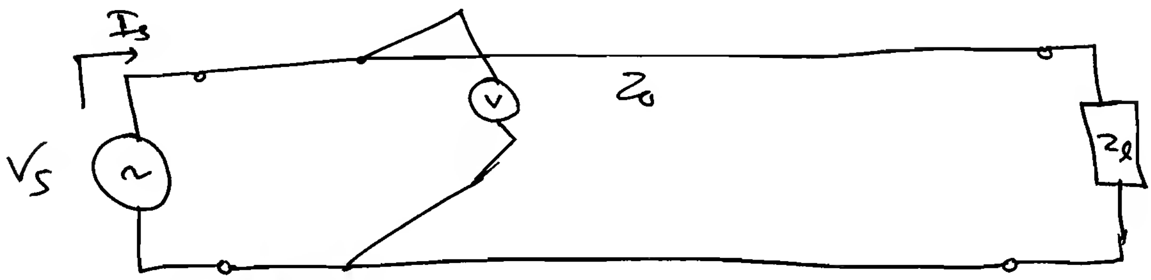
$$\therefore \frac{K_B}{K_A} = 1\angle -60^\circ.$$

$$\therefore k_B = (0.2\angle 30^\circ)(1\angle -90^\circ).$$

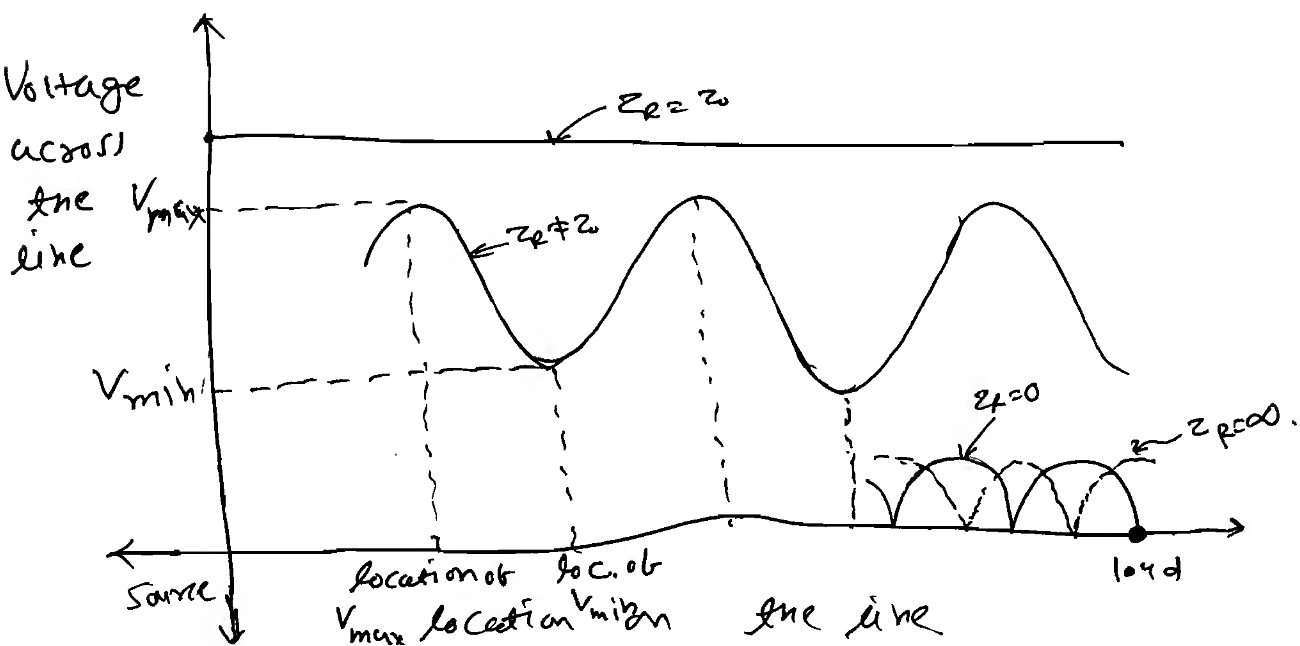
$$\therefore \boxed{K_B = 0.2\angle -30^\circ}$$

* Voltage Standing waves and Standing wave ratio:

→



$x = 0$ (or) $y = l$.



$$\boxed{V_{max} = |V_i| + |V_r|.}$$

$$\therefore \boxed{V_{min} = |V_i| - |V_r|.}$$

i: incident

r: reflected.

→ The Voltage at any point on the transmission line is the vectorical sum of incident wave voltage and reflected wave voltage at same ~~a~~ location on the transmission line this two voltages may add in in phase. when they add in in phase a voltage maximum is observed on the transmission line. when they adding in the out of phase a voltage minimum is observed on the transmission line. therefore, the voltage across a transmission line swings betn maximum voltage to the minimum voltage and vice versa.

→ Fig. shows Voltage Standing Waves for different loads

→ The successive distance betn minima to maxima (OR) maxima to minima is quarter wavelength. ($\lambda/4$).

→ The successive distance bet^n two minimas (or) two maximas is half wave length ($\lambda/2$).

→ location of $V_{max} \longleftrightarrow I_{min} \longleftrightarrow Z_{max}$.

$V_{min} \longleftrightarrow I_{max} \longleftrightarrow Z_{min}$.

* VSWR (S):

→ $$\boxed{S = \frac{V_{max}}{V_{min}}}.$$

$$\therefore = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}.$$

$$= \frac{1 + \left|\frac{V_r}{V_i}\right|}{1 - \left|\frac{V_r}{V_i}\right|}.$$

$$\therefore S = \frac{1 + |K|}{1 - |K|}.$$

where, $K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$.

$K = |K| \angle \phi$.

→ 'k' is the reflection coefficient at the load.

→ $|K|$ is the magnitude of 'k'.

ϕ is the phase of 'k'.

when $Z_R =$	K	$\|K\|$	S
Z_0	0	0	1
$0\,\Omega$ (SC)	-1 $1\angle 180^\circ$	1	∞
$\infty\,\Omega$ (OC)	1 $= 1\angle 0^\circ$	1	∞.

$|K|_{max} = 1$, $|K|_{min} = 0$.

$S_{max} = \infty$, $S_{min} = 1$.

→ It can be proved that

$Z_{max} = SZ_0$ ✓

$Z_{min} = \frac{Z_0}{S}$ ✓

→ If 'K' is the reflection coefficient at the load then $|K|^2$ is called power refl. coeff.

→ % of power reflected $= 100\,|K|^2$ %.

% of power delivered $= (1 - |K|^2)\,100$ %.

→ Return loss $= 20 \log |K|$ dB.

$=$ decibel value of $|K|^2$ ✓

∴ Return loss $= 10 \log_{10} |K|^2$.

$= 20 \log_{10} |K|$.

* Location of V_{max}

$2\beta y_{max} - \phi = 2n\pi$

$n=0 \rightarrow 1^{st}$ max.

$n=1 \rightarrow 2^{nd}$ max.

$\vdots \qquad \vdots$

Location of V_{min}

$2\beta y_{min} - \phi = (2n+1)\pi$

$n=0 \rightarrow 1^{st}$ min

$n=1 \rightarrow 2^{nd}$ min.

$\vdots \qquad \vdots$

Ex-1 A transmission line is terminated by ~~free~~ pure reactance. Assume characteristics impedance of the line is $Z_0\ \Omega$ (pure real). Find magnitude of the reflection coefficient and voltage standing wave ratio.

Ans:

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$\therefore \ |K| = 1.$

$$S = \frac{1+|K|}{1-|K|} = \frac{1+1}{1-1}$$

$\therefore \ \boxed{S = \infty}$

Power reflection coefficient $|K|^2 = 1.$

Ex 1 A transmission line is terminated by 100 Ω. Assume Characterstic impedence of the line is ~~100~~ 50 Ω. The incidence power is 10W. Find the power delivered to the load, the amount of power Reflected. Assume the line is lossless.

Ans: $Z_R = 100\,\Omega$, $Z_0 = 50\,\Omega$

$$\therefore \quad K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{100-50}{100+50}$$

$$\therefore \quad K = \frac{50}{150} = 1/3.$$

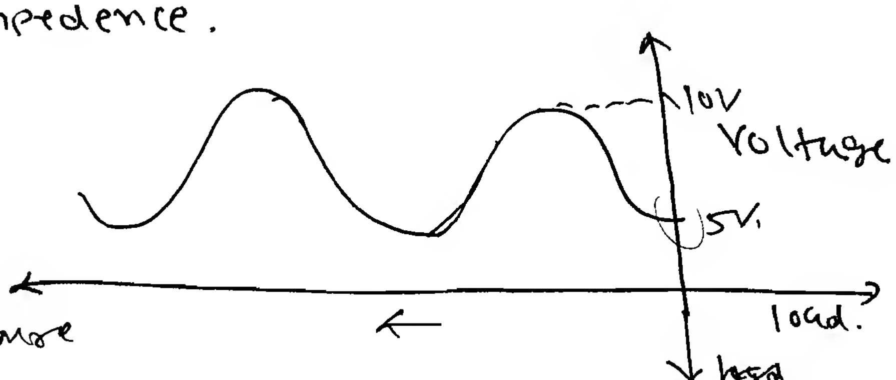
$$\therefore \quad |K|^2 = \frac{1}{9}.$$

power incident = 10W.

power reflected $= \frac{1}{9}(10) = 1.11\,W.$

power delivered $= \left(1 - \frac{1}{9}\right) 10 = 8.89$ watts.

Ex 2 figure shows that A Voltage standing wave pattern of a loss-less transmission line. assume Characteristic impedence of the line 100 Ω. Find S, K and load impedence.



$\therefore$ $V_{max} = 10V.$

$V_{min} = 5V.$

$$\therefore \quad S = \left|\frac{V_{max}}{V_{max}}\right| \frac{\cancel{V_{min}}}{+\cancel{V_{min}}}.$$

$$\therefore \quad S = \frac{10}{5} = 2$$

$\boxed{S = 2}$

$$\therefore \quad |K| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = \frac{1}{3}.$$

$$\therefore \quad K = |K|\angle\phi.$$

$$\therefore \quad K = \frac{1}{3}\angle\phi.$$

$n=0$ for 1^{st} min.

$$\therefore \quad 2\beta y_{min_1} - \phi = (2n+1)\pi.$$

$$\therefore \quad 2\left(\frac{2\pi}{\lambda}\right)(0) - \phi = \pi.$$

$y_{min_1} = 0$ (i.e.) at the load itself.

$$\phi = \pi.$$

$$\therefore \quad K = \frac{1}{3}\angle 180 = -\frac{1}{3}.$$

$$\therefore \boxed{K = -\frac{1}{3}.}$$

$$\therefore \quad \frac{Z_R}{Z_0} = \frac{1+K}{1-K} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}}.$$

$$\therefore \quad Z_R = \frac{1}{2} \times Z_0$$

$$\therefore \quad Z_R = \frac{100}{2} \Rightarrow \boxed{Z_R = 50\,\Omega}$$

Ex 3 In the above problem find min impedence and max impedence in transmission line.

Ans:

$$Z_{max} = S Z_0$$

$$Z_{min} = Z_0 / S$$

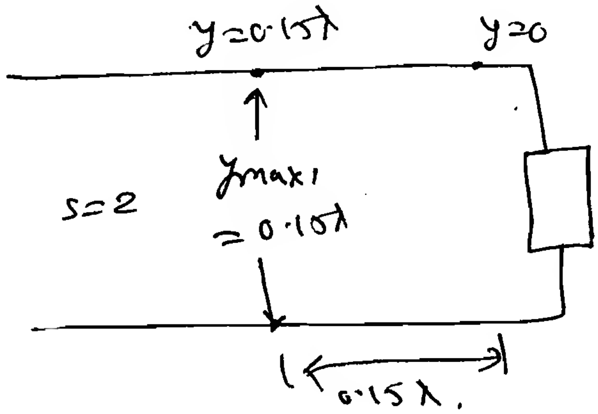
$$\therefore Z_{max} = 2(100) = 200\Omega.$$

$$Z_{min} = \frac{100}{2} = 50\Omega.$$

Ex 4 A transmission line is terminated by an unknown load impedence. The voltage maximum is observed in a distance of 0.15λ from the load. The VSWR measured on the line is ~~to 20~~ 2.0. Find the normalized load impedence.

(**Hint:** If any impedence is divided with Z_0 then that impedence is known as normalized impedence)

Ans:



$$\therefore |\bar{\Gamma}| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = 1/3.$$

$\therefore \quad 2\beta y_{max1} - \phi = 2n\pi$. $\quad n=0$ for 1st max.

$$\therefore \quad 2 \times \frac{2\pi}{\lambda} \times 0.15\lambda - \phi = 0$$

$$\therefore \quad \boxed{\phi = 108^\circ}$$

$$\therefore \quad \boxed{K = 0.333 \angle 108^\circ}$$

$$\therefore \quad \frac{Z_R}{Z_0} = \text{Normalized impedence} = \frac{1+K}{1-K}.$$

$$=$$

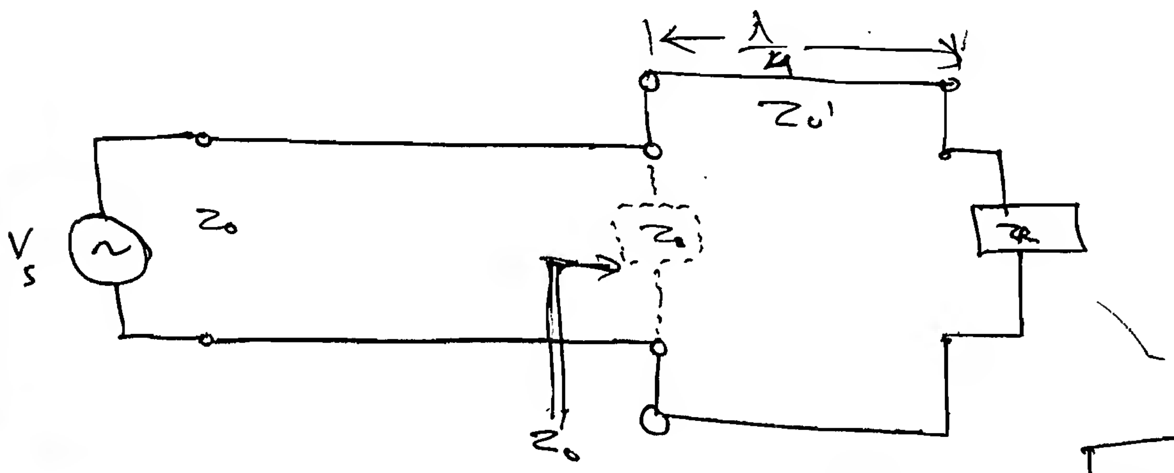
$$\therefore \quad \boxed{\frac{Z_R}{Z_0} = \frac{1 + 0.333\angle 108^\circ}{1 - 0.333\angle 108^\circ}}.$$

☆ Impedence Matching.

→ When $Z_R \neq Z_0$ reflection would results this reflection will travell towards the Source. The Purpose of a Source is to deliver the energy because of the reflection the Source is bound to ~~ex~~ accept the reflection. Due to this some Source leads to unstable (OR) they may leads to damage. Impedence matching technique is employed for protecting the Sources from the reflections.

(1) Quaterwave transformer:

→ Connecting a Quaterwave line between the main transmission line and the load impedence is shown in the figure. Characteresticy Impedence of this Quater wave line is the geometric mean of Z_0 and Z_R



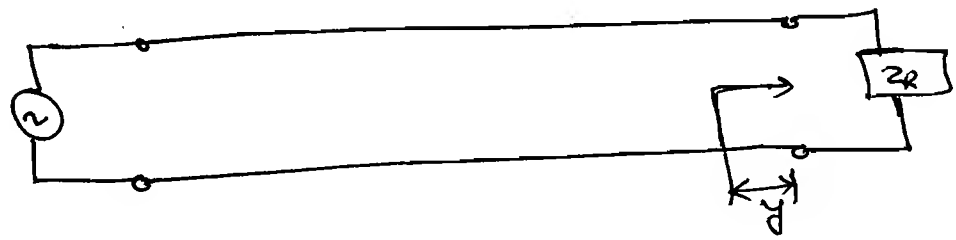
$$Z_0 = \frac{Z_0'^2}{Z_R} \Rightarrow \boxed{Z_0' = \sqrt{Z_0 \cdot Z_R}}$$

→ This technique is very simple. It has disadvantge of whenever the freq. of operation change length of the quater. wave line has to readjested by disconnecting from the main transmission line.

② Stub matching:

→ A Section of a lossless transmission line either it is open circuited (or) short circuited can act as a circuit reactive elements (or) circuit susseptive elements desired reactance (or) susseptance can be achieved by properly choosing length of the transmission line. This section are named as stubs. and are used in the impedence matching techniques. Hence, the name stub matching.

⇒



$$Z_y = Z_0 \frac{Z_R + j Z_0 \tan\beta y}{Z_0 + j Z_R \tan\beta y} \, \Omega.$$

Normalized Imp $Z_{yn} = \frac{Z_y}{Z_0}$

$$\therefore Z_{yn} = \frac{Z_R + j Z_0 \tan\beta y}{Z_0 + j Z_R \tan\beta y}.$$

$\therefore$ Normalized admitance $Y_{yn} = \frac{1}{Z_{yn}} = \frac{Z_0 + j Z_R \tan\beta y}{Z_R + j Z_R \tan\beta y}$

If $\boxed{Z_{yn} = 1}$ $\Rightarrow$ $Z_y = Z_0$ $\Rightarrow$ line is matched.

If $\boxed{Y_{yn} = 1}$

$$\Rightarrow \quad Y_{yn} = \frac{Z_0 + jZ_R \tan\beta y}{Z_R + jZ_0 \tan\beta y}.$$

By rationalizing, the real and im. parts can be separated.

$\therefore$ Y_{yn} = real part + j Im. part..

$\rightarrow$ real part $\rightarrow$ function of $(\check{Z}_R, \check{Z}_0, \check{\beta}, \overset{(?)}{y}) = 1$

Im. part $\rightarrow$ function of $(\check{Z}_R, \check{Z}_0, \check{\beta}, y) = ?$

Q1. Can we find at what 'y', the real part becomes unity?

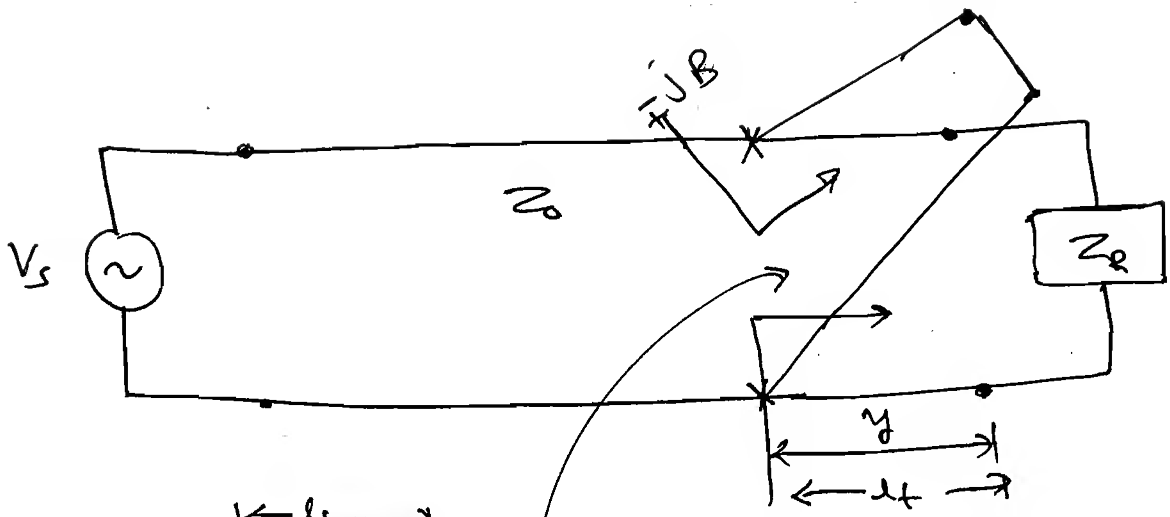
Ans: yes.

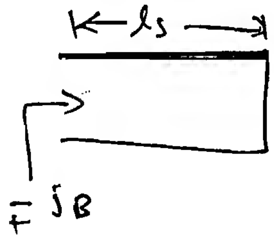
Q2: ~~from~~ At the above y, can we find the value of Im. part?

Ans: yes.

$\Rightarrow$ Some how we are calculated above y. At that y Normalized admittance,

$\therefore$ $\boxed{Y_{yn} = 1 + jB}$ $\Rightarrow$ B = Susseptance.





$Y_{yn} = 1 + jB.$

Resultant admittance

$Y_{yn} = 1 \pm jB \mp jB = 1.$

$\therefore \boxed{Y_{yn} = 1} \rightarrow$ $\boxed{\text{line is matched.}}$

$\therefore Y_{yn}$

→ We have to find 'l_s' and l_t.

$$\therefore \quad l_s = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{S}}{S-1}.$$

$$l_t = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta}.$$

l_s : length the S.C. stub.

l_t : location of the stub from the load.

* SMITH CHART:

→ It is a graphical solution provider for any transmission line problems.

It is a family of circles.

→ $$K = \frac{Z_R - Z_0}{Z_R + Z_0}.$$

$$\therefore \frac{Z_R}{Z_0} = \frac{1+K}{1-K}.$$

let $$Z_r = \frac{Z_R}{Z_0} = \frac{1+K}{1-K}, \quad Z_r = \text{No units.}$$
$$= R + jx$$

'R' : Real part of Z_R.

'x' : Im. part of Z_R.

→ $$K = |K| \angle \phi.$$
$$K = K_r + j K_x.$$

K_r : real (K).

K_x = Img. (K).

$$R + jx = \frac{(1+K_r + jK_x)(1-K_r + jK_x)}{(1-K_r - jK_x)(1-K_r + jK_x)}.$$

→ By rationalizing, one can separate real and Im. parts.

$$R = \frac{1 - K_r^2 - K_x^2}{(1-K_r)^2 + K_x^2} \quad — (1).$$

$$\therefore \quad X = \frac{2k_x}{(1-k_r)^2 + k_x^2} \quad (2).$$

→ Eqⁿ -(1) & (2) represents a family of circles.

Consider eqⁿ -(1).

$$R = \frac{1 - k_r^2 - k_x^2}{(1-k_r)^2 + k_x^2}.$$

Cross multiply..

$$\therefore R k_r^2 - 2R k_r + R + R k_x^2 = 1 - k_r^2 - k_x^2.$$

$$\therefore k_r^2 (1+R) - 2 k_r R + k_x^2 (1+R) = 1-R.$$

divide by $(1+R)$.

$$\therefore k_r^2 - \frac{2R}{(1+R)} . k_r + k_x^2 = \frac{1-R}{1+R}.$$

add $\left(\frac{R}{1+R}\right)^2$ on both sides.

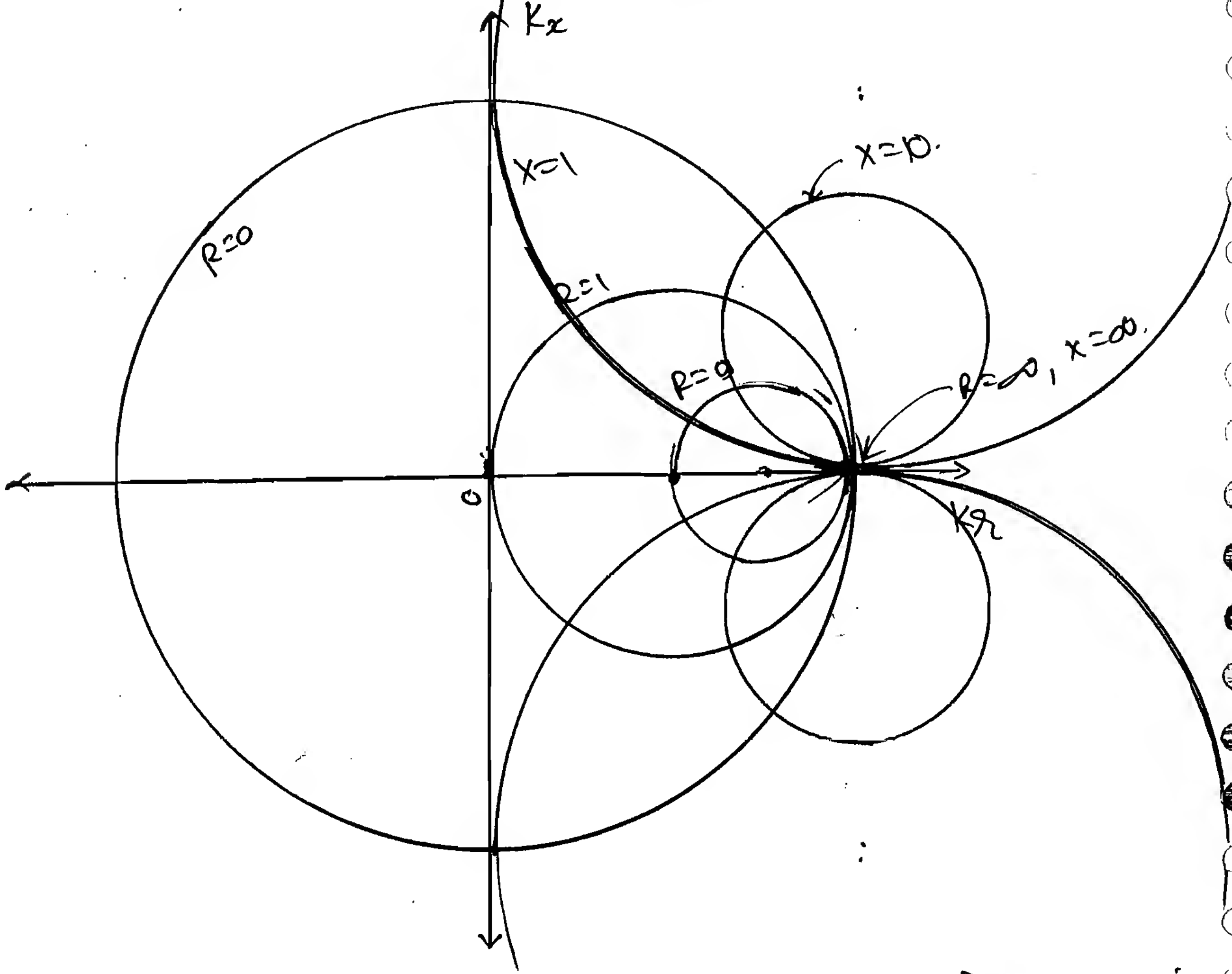
$$\left(k_r - \frac{R}{1+R}\right)^2 + (k_x - 0)^2 = \left(\frac{1}{1+R}\right)^2.$$

→ The above eqⁿ Represents eqⁿ of a circle with centre $\left(\frac{R}{1+R}, 0\right)$ and radius $\left(\frac{1}{1+R}\right)$ on (k_r, k_x) axies.

→ R can assume all possible values ranging from 0 to ∞. Therefore, one can draw

infinite number of circles

→ As a value of R is increasing the radius of a circle is decreasing.



* All constant R circles having following properties.

(i) All the circles are passing through (1,0)

(ii) All the circles are having their centres betn (0,1) on Kr axis.

(iii) All the circles are neither concentric nor cutting each other.

→ consider eqn (4)

$$X = \frac{2k_x}{(1-k_r)^2 + k_x^2}.$$

$\therefore \; k_r^2 X - 2Xk_r + X + k_x^2 = 2k_x.$

divide with X.

$\therefore \; k_r^2 - 2k_r + 1 + \frac{k_x^2}{X} = \frac{2k_x}{X}.$

add $\left(\frac{1}{X}\right)^2$ on both sides.

$$(k_r - 1)^2 + \left(k_x - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2.$$

→ The above eqn represents eqn of a circle with centre $(1, 1/X)$ and radius $\frac{1}{X}$. and radius $1/X$. on k_r, k_x axies.

→ X can assume all possible values ranging from $(-\infty, +\infty)$ therefore infinite no. of circles can be drawn

→ $X \rightarrow -\infty$ to $+\infty$.

X=1
(1,1), 1

X=10
(1, 0.1), 0.1

X=-1
(1,-1), 1

X=0
$(1, \infty), \infty$

⇒ As x increases ⇒ Radius is decreasing:

→ All constant x circles are having following properties

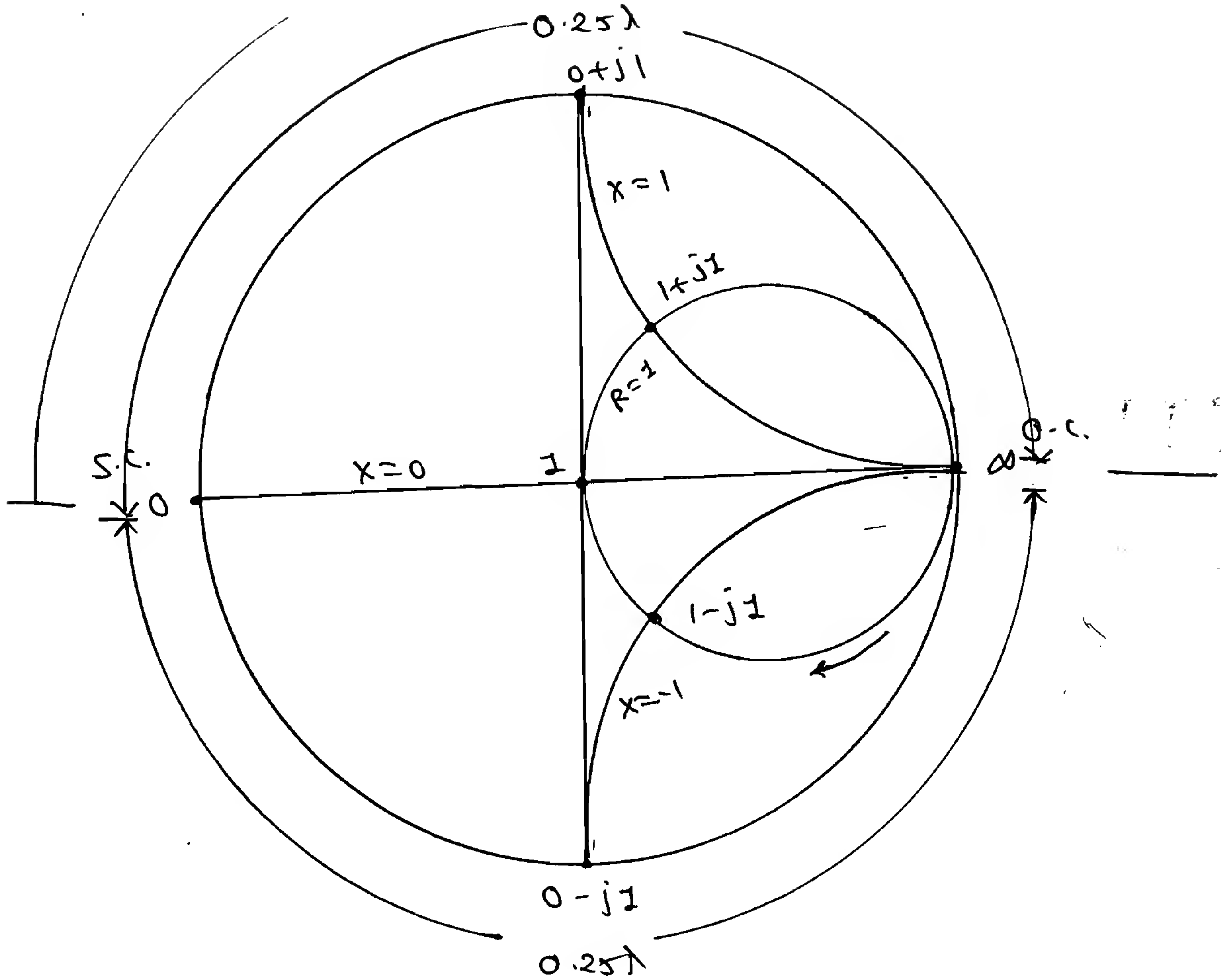
(1) All the circles are having there centres on $k_r = 1$ line

(2) All the circles are passing through (1, 0).

(3) The circles above the horizontal axis represents locus of +ve reactance whereas below the horizontal axis represents locus of (-ve) Reactance.

(4) All the circles are neither concentric nor cutting each other.

(5) The locus of constant -R circles and constant X- circles together is called Smith Chart

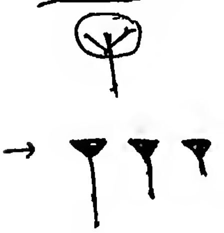


→ One complete revolution of the smith chart indicates a distance of λ/2.

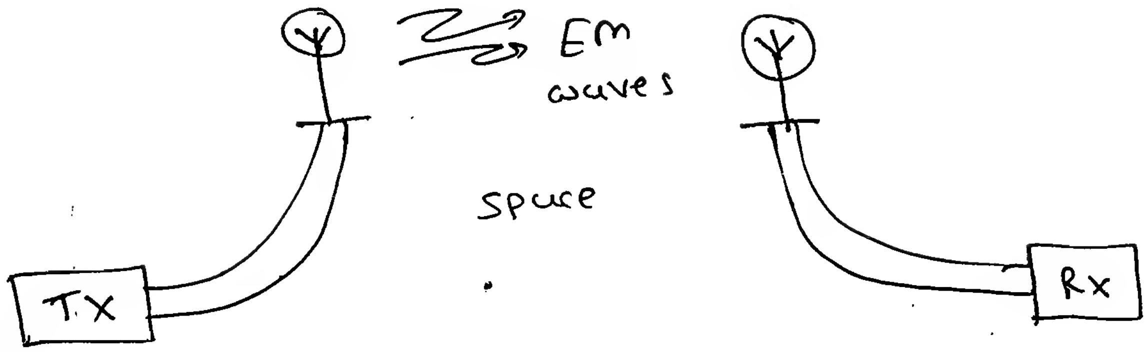
→ ~~this~~ on a constant R circle if we are moving in the clockwise direction we are adding an inductive reactance in series to the impedence.

✫ Basics of Antenna:

Symbol:



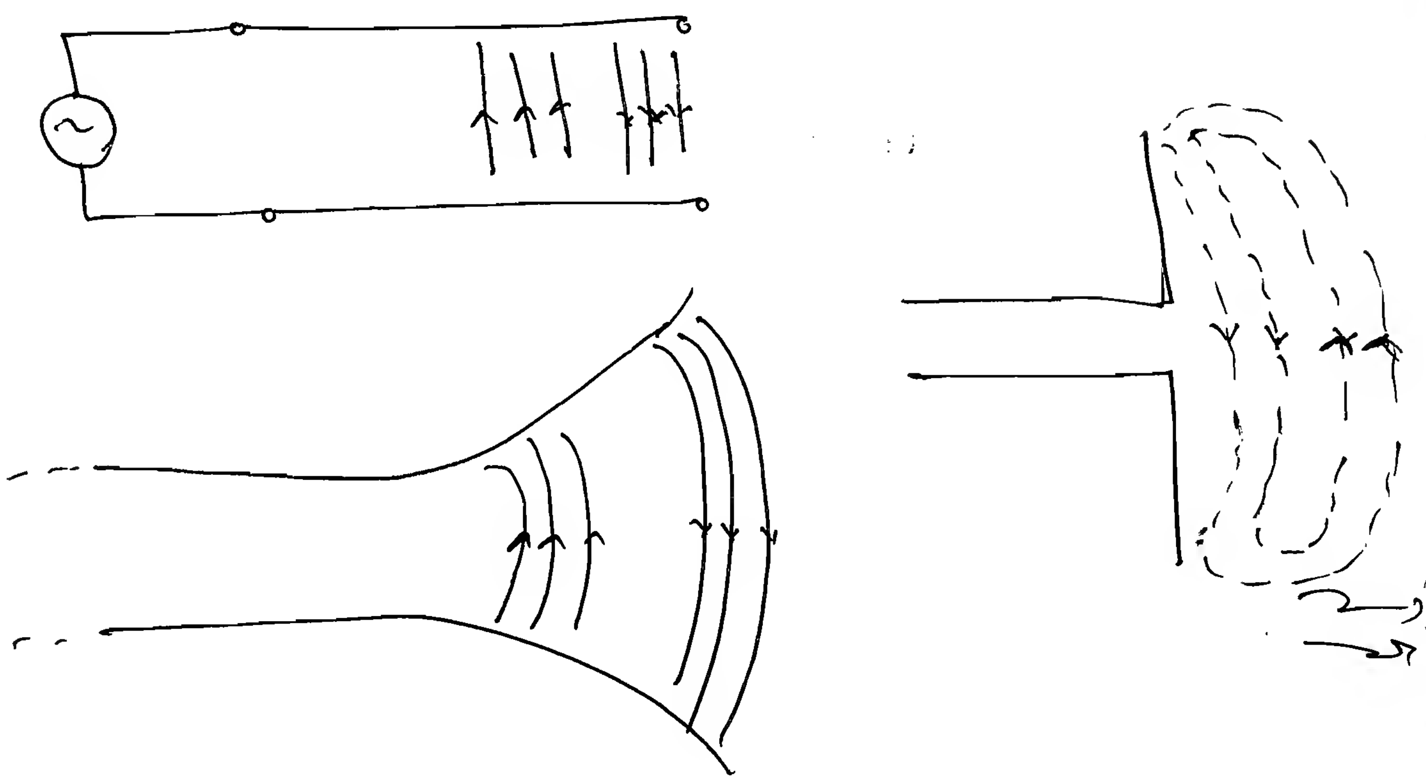
signal strength



Antenna:

- Coupling devices (B/w Tx ~~to~~ space & space to Rx).
- It radiates | Receives EM waves.
- It is tuned.
- Passive.

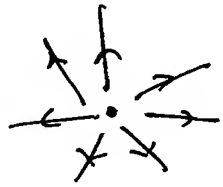
✱ Radiation mechanism:



* Isotropic Radiator:

→ It is an impractical antenna (fictisious). It is capable of radiating, receiving uniformly in all directions.

e.g.(i) point source.



* Directional Radiators:

→ All practical antennas are Directional radiators. They are capable of radiating receiving em EM waves through some perticular directions.

* Omnidirectinal Radiators:

→ This is a special kind of directional radiator capable of radiating uniformly in the azimuth plane and having non-uniform radiation in the elevation plane.

e.g. dipole antenna.

⇒ By Reciprocity theorem it is proved that radiation properties of an antenna are identically same whether the antenna use for transmission purpose or reception purpose.

→ Radiation Properties include * radiation pattern.

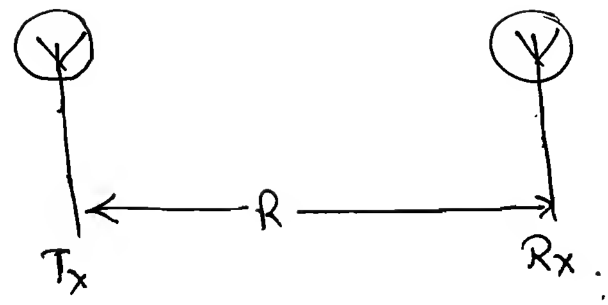
* Average Radiation density.
* Average Radiation intensity.
* Average Radiation power.
* Directive Gain.
* Directivity.
* Power gain.
* maximum power gain.
* Total efficiency of Antenna.
* effective aperture area.
* Antenna polarization. etc.
*
=> These are the major things of antenna.
=> These are the antenna parameters.

(1) Radiation Pattern:

→ It is the locus of received field strength (OR) power at a fixed far distance as a function of Space co-ordinates if the received quantity is field strength then it is called field strength pattern.

→ If the received quantity is power then it is called power pattern.

$\Rightarrow$



* Fraunhoffer Region of field:

→ If $\boxed{R > \frac{2D^2}{\lambda}}$ ⇒ Fraunhoffer far field zone

⇒ The fields in this zone are called active fields.

$E \propto \frac{1}{r}$

⇒ These are useful for radiation purpose.

→ If $\boxed{R < \frac{2D^2}{\lambda}}$ ⇒ Fraunhoffer near field zone.

⇒ The fields in this zone are called reactive fields.

$E \propto \frac{1}{r^2}$

⇒ They are not useful for radiation purpose.

D: max. dimension of the antenna.

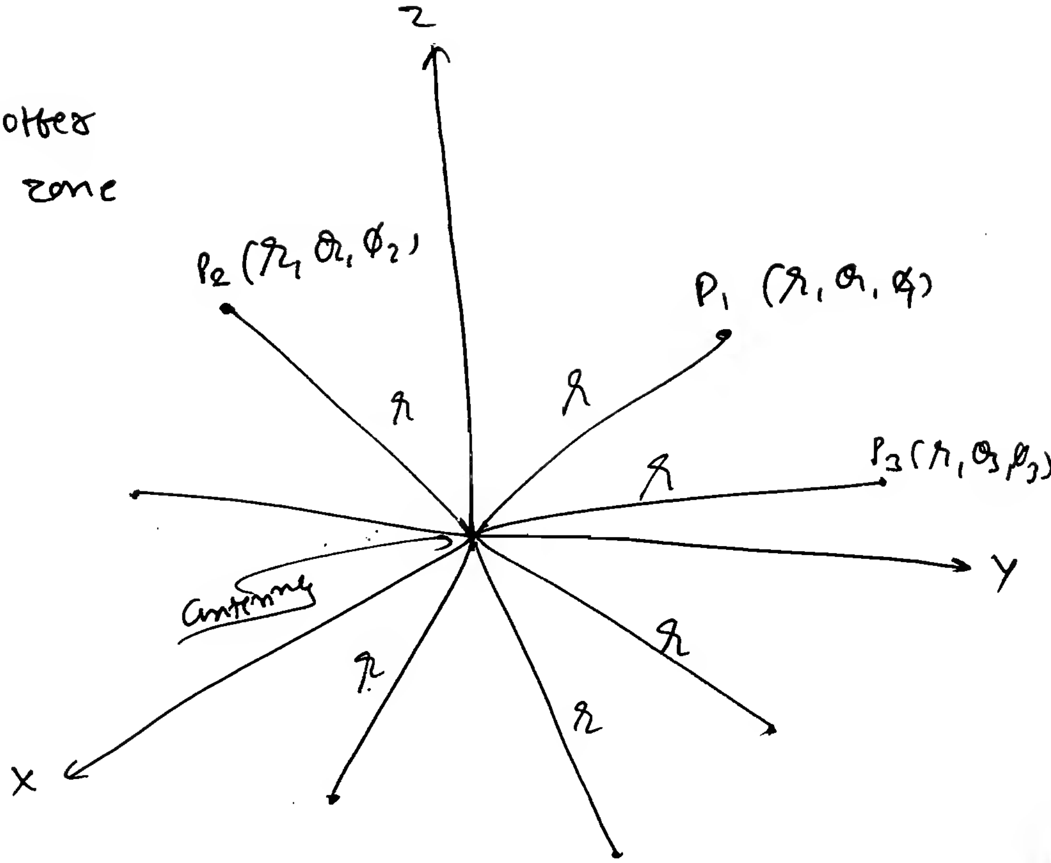
λ: operating wavelength.

→ All the antennas are intended to be operated in the Fraunhoffer far field zone only.

*

Satisfies fraunhoffer far field zone

$\boxed{r = \text{constant}}$



$r = \text{const.}$

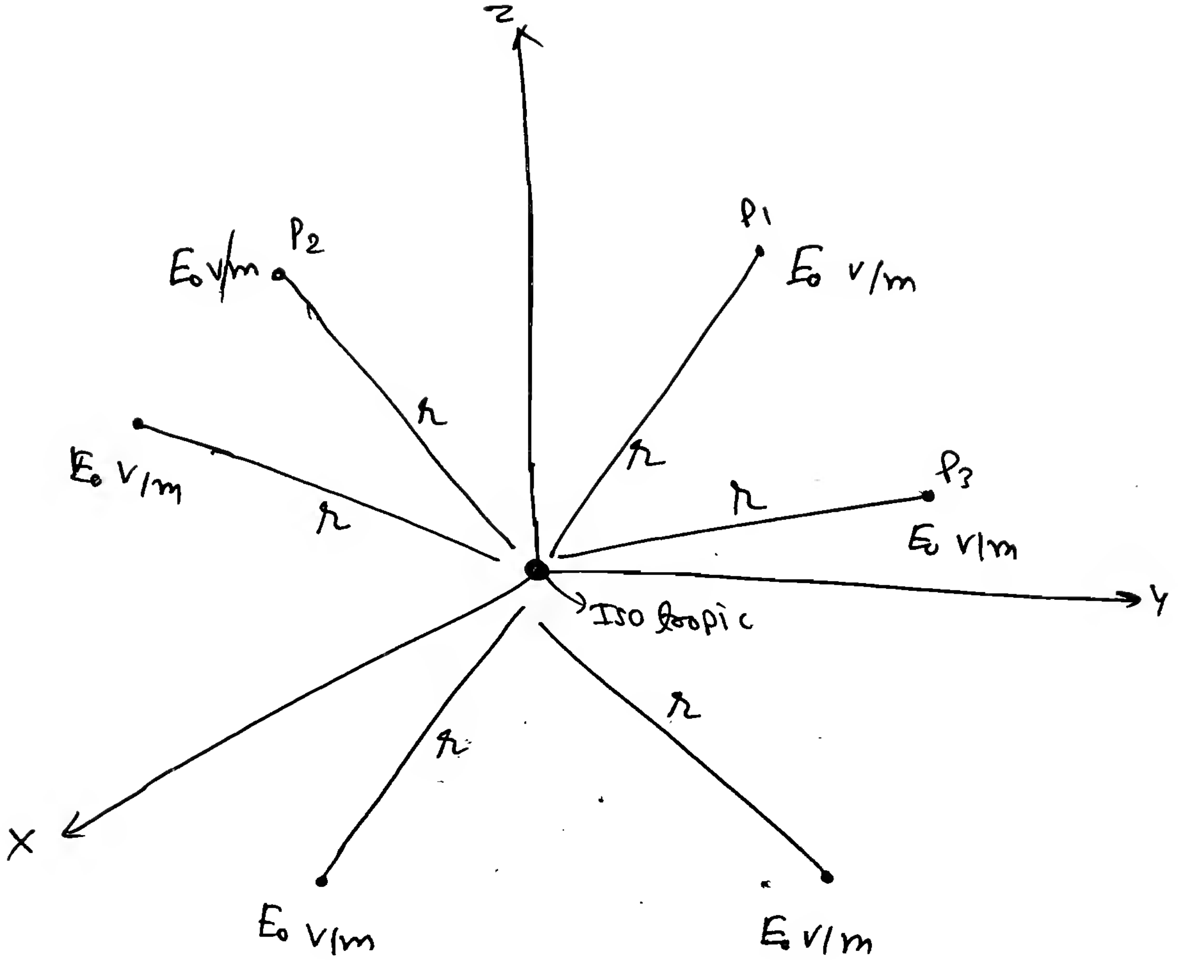
$\theta \Rightarrow$	0 to 180
$\phi \Rightarrow$	0 to 360

→ We assume that, so the antenna positioned at the origin. Around the antenna at a fixed distance r one can have infinite no. of points. All those points are having equal distance from the origin and are distinguishible.

* Radiation pattern of an Isotropic Radiator:

→ we assume that isotropic radiator is positioned at the origin. at a fixed distance in the fraunhoffer field zone we observed E_0 v/m at various point. i.e.

$r = \text{const.}$
$\theta = 0 \text{ to } \pi$
$\phi = 0 \text{ to } 2\pi$



→ The three dimensional radiation pattern of an Isotropic antenna appears to like a spherical cell because it radiates uniformly in all directions.

→ The radiation pattern is a three dimensional view. Radiation pattern can also be investigated in the following two principal plane:

1. Azimuthal plane patterns ($\theta = 90°$).

2. Elevation plane patterns ($\phi = $ const).

→ we consider $\phi = 0°$ and $\phi = 180°$.

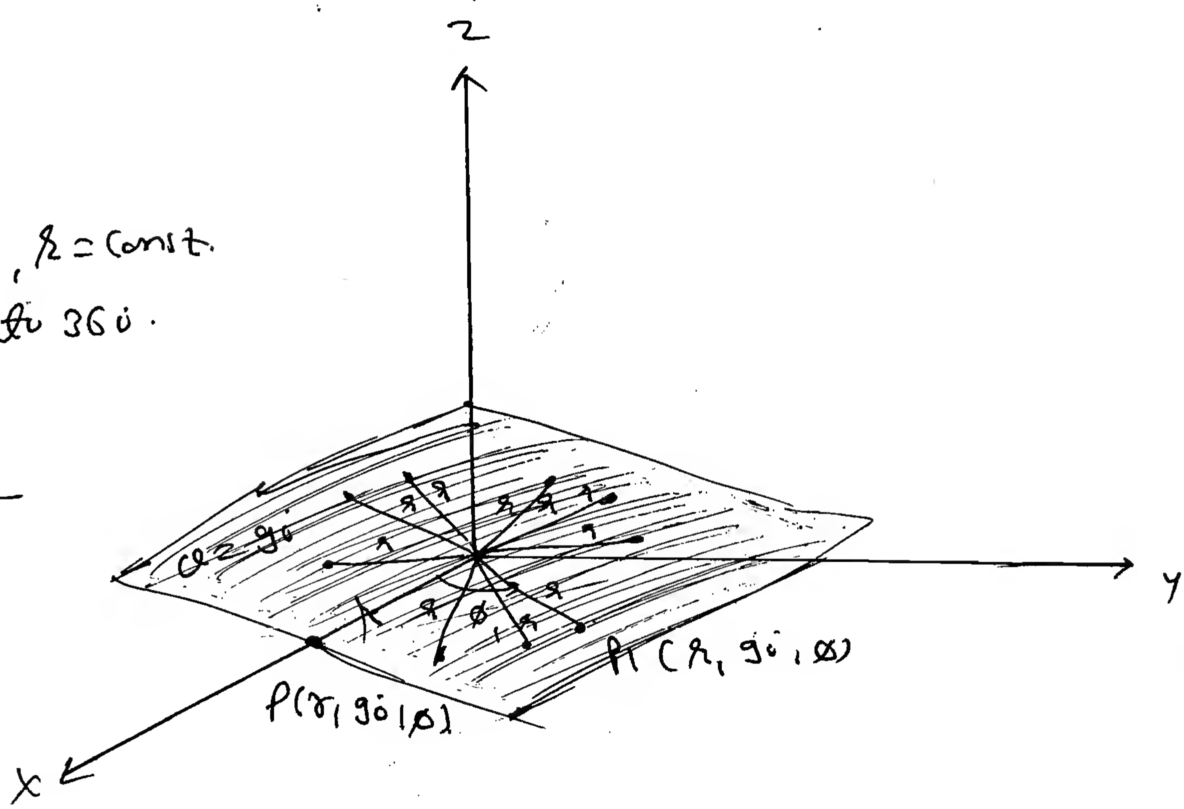
→ Azimuthal plane pattern of a Radiator.

* Azimuthal Plane Pattern of an Isotropic Radiator.

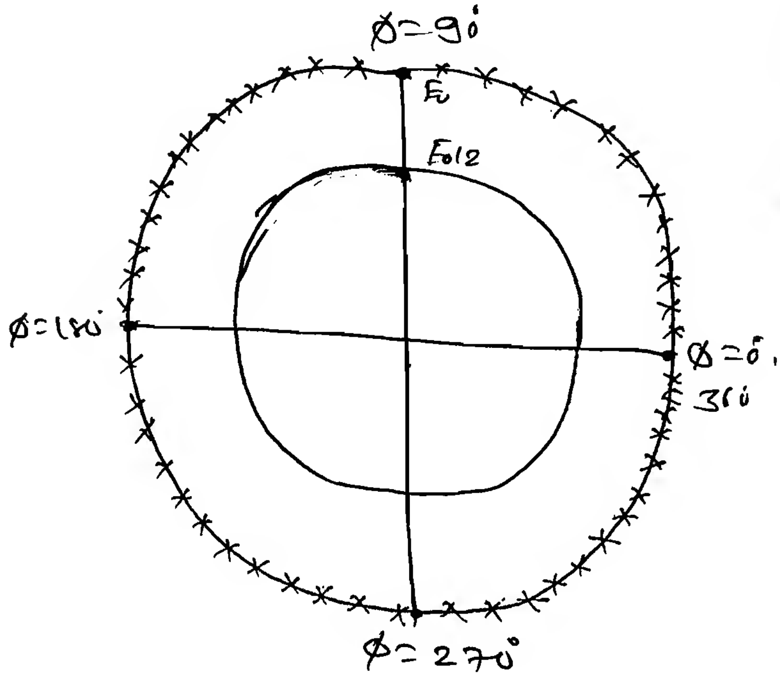
$\theta = 90^\circ$

$\theta = 90^\circ$, $r = $ Const.

$\phi \to 0$ to 360°.

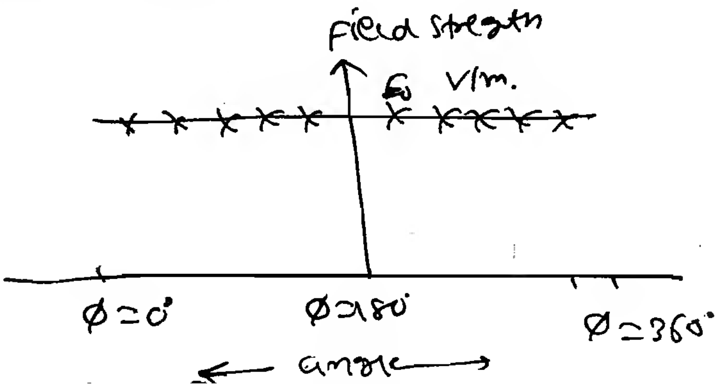


$\phi =$	Field strength
0°	E_0 V/m
1°	E_0 V/m
2°	E_0 V/m
⋮	⋮
20°	E_0 V/m
⋮	⋮
360°	E_0 V/m



Polar plot

* Linear plot:

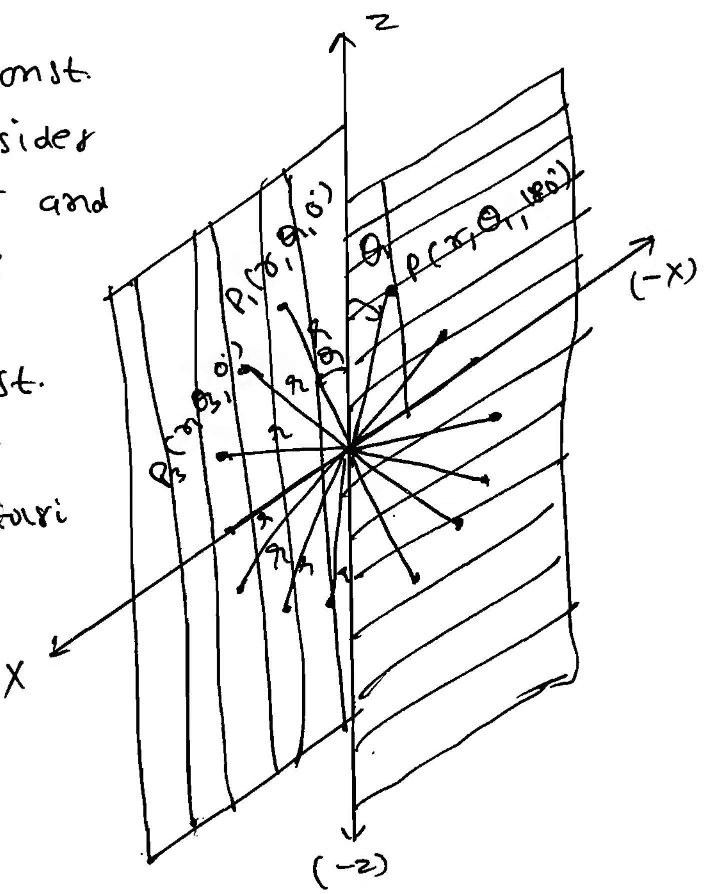


→ circular locus on the polar plot and horizontal straight line locus on the linear plot indicates that Isotropic radiator radiates uniformly in all directions in the azimuthal plane.

* Elevation plane pattern of a Isotropic radiator.

ϕ = const.
we consider
$\phi = 0^\circ$ and
$\phi = 180^\circ$

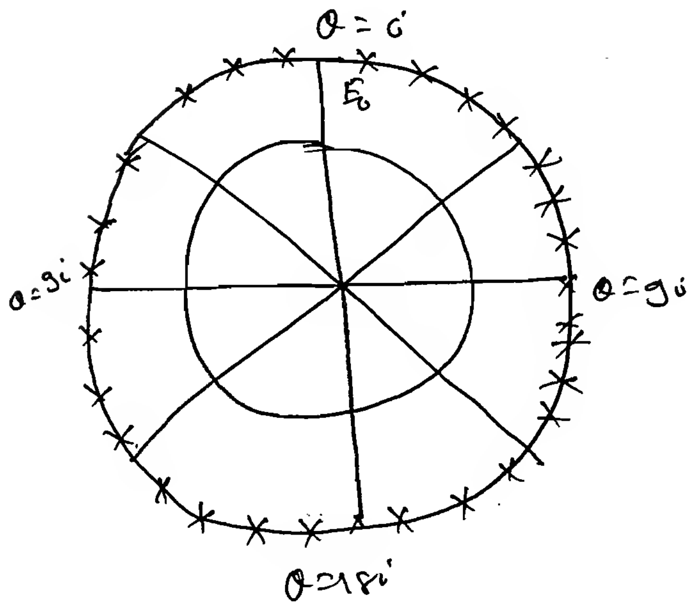
r = const.
$\phi = 0^\circ$
$\theta \to 0$ to 180°

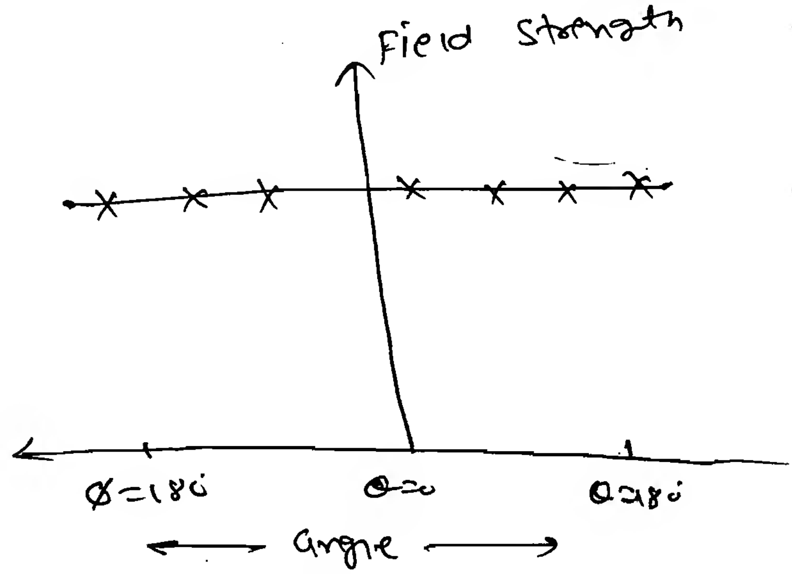


θ	Field strength	
0°	E_0 V/m	$\phi = 0^\circ$
1°	E_0 V/m	r
2°	E V/m	=
⋮	⋮	const.
180	E_0 V/m	

$\theta =$	Field strength	
0°	E_0	$\phi = 180^\circ$
1°	E_0	r
2°	⋮	=
⋮		const.
180°	E_0	

Polar plot

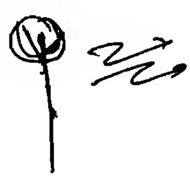




→ circular locus on the polar plot and horizontal straight line locus on the linear plot indicates that isotropic radiators radiates uniformly in all directions in the elevation plane.

(2) Average Radiation Density:

→ It is defined as the avg. power radiated per unit area



$\bar{E}_s, \bar{H}_s$

→ If $\bar{E}_s, \bar{H}_s$ are radiated fields in the Fraunhoffer far field zone.

complex conjugated.

$$\therefore \boxed{\bar{P}_{rad} = \frac{1}{2} \bar{E}_s \times \bar{H}_s^*} \quad W/m^2$$

Avg power Radiated

$$\boxed{W_{rad} = \oint_S \bar{P}_{avg} \cdot d\bar{s} \quad \text{Watts.}}$$

$d\bar{s}$: Vector diff. surface element.

$d\bar{s}$: $r^2 \sin\theta \, d\theta d\phi \, \hat{a}_r$.

Ex 1

⇒ The avg. radiation density of an Antenna in the radial direction is given by

$$\bar{P}_{rad} = \frac{A_0 \sin\theta}{r^2}\,\hat{a}_r \quad W/m^2.$$

Where, A_0 = Constant

r, θ : Spherical Coordinates

Find avg. radiated power.

Ans: The given $\bar{P}_{rad}$ will belongs to an Antenna which is omni directional. It has uniform radiation in the azimuthal plane and non-uniform radiations in the elevation plane.

$$\bar{P}_{rad} \cdot d\bar{s} = \frac{A_0 \sin\theta}{\cancel{r^2}} \times \cancel{r^2} \sin\theta \cdot d\theta\, d\phi.$$

$$= A_0 \sin^2\theta \cdot d\theta\, d\phi.$$

$$\therefore \quad W_{rad} = \int_{(\theta)} \int_{(\phi)} \bar{P}_{rad} \cdot d\bar{s}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{380^\circ} A_0 \sin^2\theta \cdot d\theta\, d\phi.$$

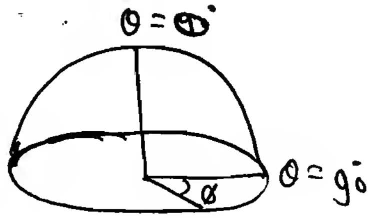
$$= A_0 \times [2\pi] \times \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$= A_0 \times \cancel{2}\pi \times \left[\frac{\pi}{\cancel{2}}\right]$$

$$\therefore \quad \boxed{W_{rad} = \pi^2 A_0 \text{ watts}}$$

Ex 2
→ Repeat the above example to calculate avg. power radiated in the upper hemisphere.

Ans: Upper hemi sphere



θ = 0 to 90°

φ = 0 to 360°.

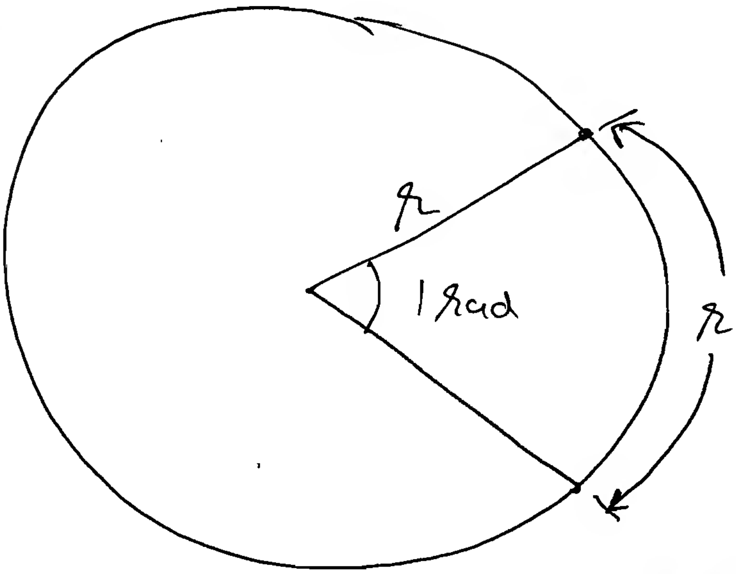
$$\therefore W_{rad} = \int_0^{2\pi}\int_0^{\pi/2} A_0 \left[\frac{1 - \cos 2\theta}{2}\right] d\theta\, d\phi.$$

$$= A_0 \times [2\pi] \times \left[\frac{\theta}{2} - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}.$$

$$= A_0 \times 2\pi \times \left[\frac{\pi}{4}\right]$$

$$\therefore \boxed{W_{rad} = \frac{1}{2}\pi^2 A_0 \text{ Watts.}}$$

* Radian:



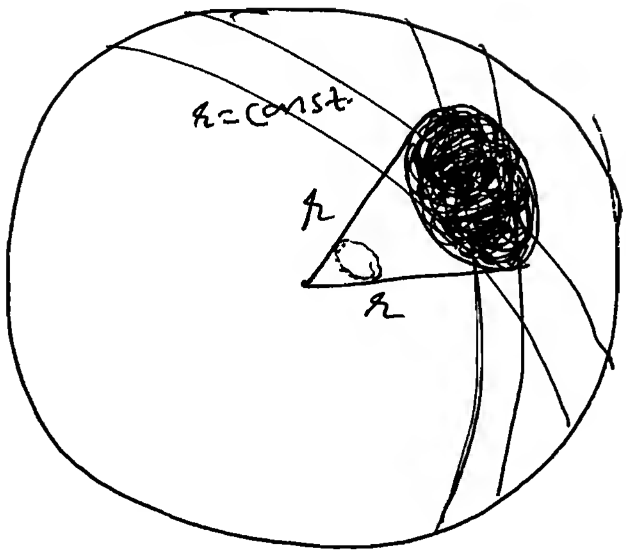
r = 1 rad.

$2\pi r = \frac{2\pi r}{r} \times 1$

$= 2\pi$ radian

The total angle substended at the centre of a circle is 2π rad.

* Steradian:



$r^2 \rightarrow 1$ steradian

$4\pi r^2 \rightarrow \frac{4\pi r^2}{r^2} \times 1 = 4\pi$ st.

→ The total solid angle subtended at the centre of a sphere = 4π st.

(3) Avg. Radiation Intensity : $\bar{U}$

→ It is defined as average power radiated per unit solid angle.

$$\boxed{\bar{U} = r^2 \, \bar{P}_{rad} \;\; W/st.}$$

Ex 1 The average radiation intensity of an antenna in the radial direction is proportional to $\cos^8\theta$. assume the proportionality constant is unity. find the average radiated power.

Ans: $\bar{U} = r^2 \, \bar{P}_{rad}$ W/st.

$|\bar{U}| \propto \cos^8\theta$.

$$|\bar{U}| = \cos^8\theta \cdot \quad \Rightarrow \quad \bar{U} = \cos^8\theta \ \hat{a}_r \ \text{w.r.t}$$

$$\therefore \bar{P}_{rad} = \frac{\bar{U}}{r^2}$$

$$\therefore \bar{P}_{rad} = \frac{\cos^8\theta}{r^2} \hat{a}_r .$$

$$\therefore \overline{ds} = r^2 \sin\theta \cdot d\theta d\phi \ \hat{a}_r$$

$$W_{rad} = \oint_s \bar{P}_{rad} \cdot d\bar{s}$$

$$= \int_0^{2\pi}\int_0^{\pi} \cos^8\theta \cdot \sin\theta \cdot d\theta d\phi .$$

$$= + \left[\frac{t^9}{9}\right]_{-1}^{+1} [2\pi]$$

$$= \left[\frac{t^9}{9}\right]_{-1}^{1} [2\pi].$$

$$= \left(\frac{1+1}{9}\right) 2\pi$$

$$\therefore \boxed{W_{rad} = \frac{4\pi}{9} \text{ Watts.}}$$

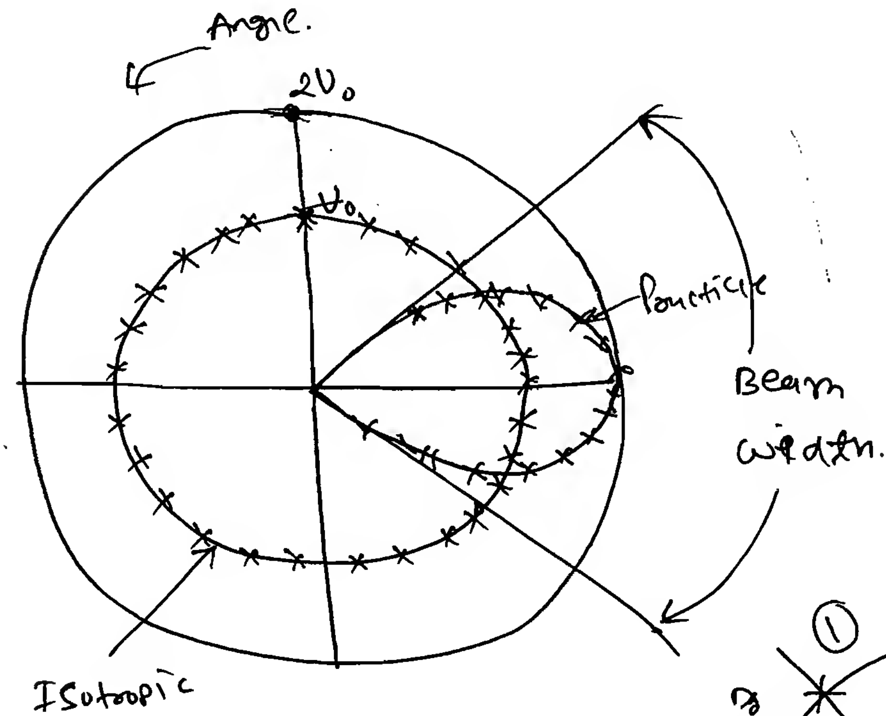
(4) Directive gain (D_g):

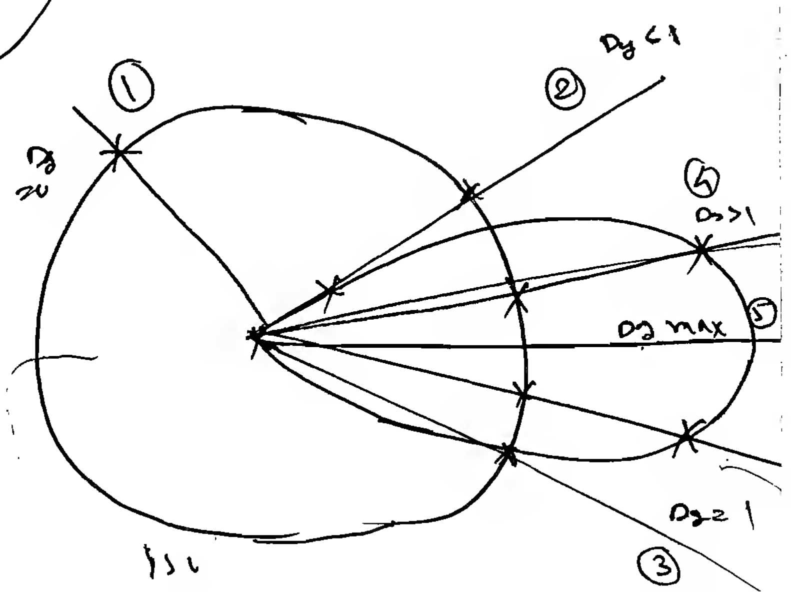
$$\Rightarrow \quad \boxed{D_g = \frac{U}{U_0}}$$

U: Practical antenna

→ Directive gain in a given direction is defined as the ratio of radiation intensity of the practical antenna whose directive gain you want to calculate to the radiation intensity of the reference antenna. The referense antenna is choosen to be Isotoopic radiator.

→ The above defination is valid under the condition that both antennas are assumed to be radiating same amount of avg. radiated powers.

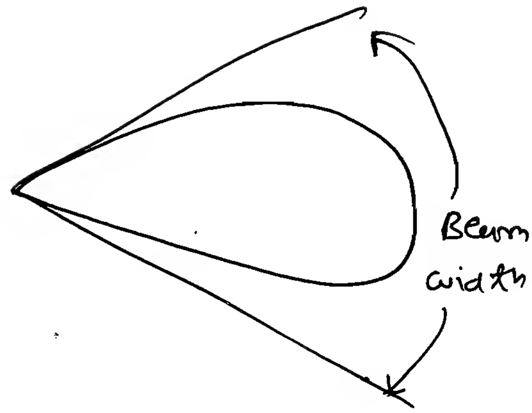


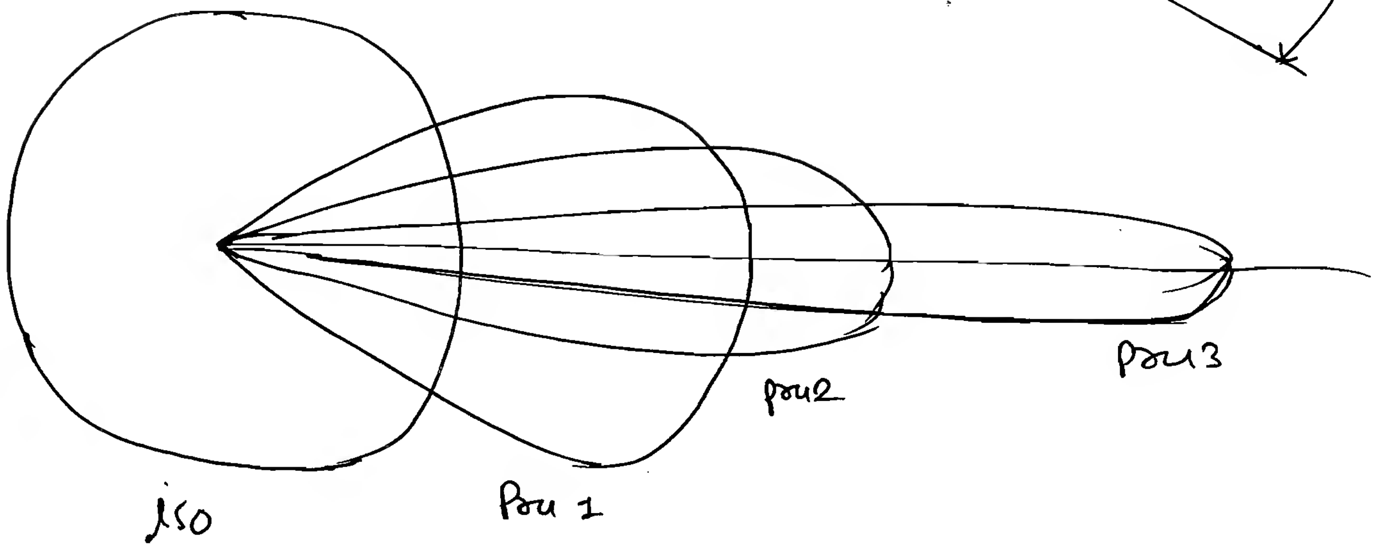


$$\omega_{rad\,(pou)} = \omega_{rad\,(iso)}.$$

$Max\,(D_g) = D_0$.

'D_0' : Directivity.





$$\omega_{rad\,(iso)} = \omega_{rad\,(pou1)} = \omega_{rad\,(pou2)} = \omega_{rad\,(pou2)} = W_{rad\,(pou3)}$$

$$(D_0)_3 > (D_0)_2 > (D_0)_1 > (D_0)_1.$$

$$D_{0\,iso} = \frac{U_0}{U_0} = 1.$$

→ As the directivity increases Beam width decreases.

→ For point to point communications the antenna and the antenna system must have high directivity. For ~~th~~

→ For the Broadcasting purposes the antenna and the antenna system must have low directivity.

$\rightarrow \quad \bar{U}_{iso} = U_0 \hat{a}_r.$

$$\bar{P}_{rad}(iso) = \frac{U_0}{r^2} \hat{a}_r.$$

$$\therefore W_{rad}(iso) = \int_{(\theta=0)}^{\pi} \int_{(\phi=0)}^{2\pi} \frac{U_0}{r^2} \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r.$$

$$= 4\pi U_0.$$

$$\therefore U_0 = \frac{W_{rad}(iso)}{4\pi} = \frac{W_{rad}(pow)}{4\pi} = \frac{W_{rad}}{4\pi}.$$

D_g

$$\therefore D_g = \frac{U}{(W_{rad}/4\pi)}.$$

$$\therefore \boxed{D_g = 4\pi \times \frac{U}{W_{rad}}}.$$

Let, $\bar{P}_{rad} = \dfrac{A_0 \sin\theta}{r^2} \hat{a}_r.$

$$\boxed{D_0 = 4\pi \times \frac{U_{max}}{W_{rad}}}$$

$$\therefore \bar{U} = r^2 \bar{P}_{rad}.$$

$$= A_0 \sin\theta \, \hat{a}_r.$$

$$\therefore U_{max} = A_0.$$

Find D_0.

$$\therefore W_{rad} = \pi^2 A_0 \text{ watts.}$$

$$\therefore D_0 = 4\pi \cdot \frac{U_{max}}{W_{rad}}.$$

$$\therefore D_0 = 4\pi \cdot \frac{\cancel{A_0}}{\pi^2 \cancel{A_0}}.$$

$$\therefore \boxed{D_0 = \frac{4}{\pi}} \qquad D_0 \text{ (in dB)} = 10 \log_{10} \left(\frac{4}{\pi}\right) \text{dB}.$$

Ex-1

→ The radiation density of an antenna in a radial direction is proportional to the $\cos^{10}\theta$ assum the antenna has radiation in the upper hemi sphere. Find the directivity.

Ans:

$$|\bar{U}| \propto \cos^{10}\theta.$$

$$\therefore |\bar{U}| = A_0 \cos^{10}\theta.$$

$$\bar{U} = A_0 \cos^{10}\theta \ \hat{a}_r. \quad \text{Where } A_0 = \text{Const.}$$

$$\therefore \bar{P}_{rad} = \frac{A_0 \cos^{10}\theta}{r^2} \hat{a}_r \ \ \text{W/m}^2.$$

$$\therefore W_{rad} = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \frac{A_0 \cdot \cos^{10}\theta}{r^2} \, r^2 \sin\theta \cdot d\theta \, d\phi.$$

$$\therefore W_{rad} = \int_{\theta=0}^{\pi/2} \int_{0}^{2\pi} A_0 \sin\theta \cdot \cos^{10}\theta \cdot d\theta \, d\phi.$$

$$= 2\pi \times A_0 \times \frac{9 \times \cancel{7} \times \cancel{5} \times \cancel{3} \times \cancel{1}}{11 \times 9 \times \cancel{7} \times \cancel{5} \times \cancel{3} \times 1} \ (1).$$

$$\therefore \boxed{W_{rad} = \frac{2\pi A_0}{11} \ \text{Watts.}}$$

$$\therefore D_0 = 4\pi \frac{U_{max}}{W_{rad}}.$$

$$= \cancel{4\pi}^{2} \frac{\cancel{A_0}}{\cancel{2\pi} \cancel{A_0}} \times 11$$

$$D_0 = 22$$

$\therefore D_0 \text{ (in dB)} = 10 \log_{20} (22) \text{ dB.}$

Ex-2

→ Radiation density of an antenna in the radial direction is given by $\sin\theta \cdot \cos^2\phi \, \hat{a}_r$. watts / steradian assume the antenna is radiating for $0 \leq \theta < \pi/2$, $0 < \phi < \pi/2$. find Directivity? Do?

(5) Power Gain:

$$\Rightarrow \boxed{G_p = 4\pi \frac{U}{W_{in}}}$$

$\rightarrow$ $W_{in} = W_{rad} + W_{loss}$.

$\rightarrow$ Max. Power Gain $G_0 = 4\pi \frac{U_{max}}{W_{in}}$.

(6) Total Efficiency of an Antenna:

$$\Rightarrow e_t = \frac{W_{rad}}{W_{in}}.$$

$$\boxed{e_t = \frac{G_0}{D_0}}.$$

(7) Effective Aperture Area (A_e)

$\Rightarrow$ It refers to physical size of the antenna.

$\rightarrow$ Larger antennas will have larger Aperture area and vice versa.

$\rightarrow$ If the antenna dimension are larger than λ then they are called large antennas and viceversa.

→ It is defined as average power receive to the average power density of the incident wave.

→ By expression,

$$\boxed{A_e = \frac{\lambda^2}{4\pi} \cdot D_g}$$

$$\therefore \boxed{A_{emax} = \frac{\lambda^2}{4\pi} \cdot D_o}$$

$\uparrow A_e \Rightarrow \uparrow D_o \Rightarrow \downarrow$ Beamwidth (narrows).

$\downarrow A_e \Rightarrow \downarrow D_o \Rightarrow \uparrow$ Beamwidth (wider).

(8) <u>Antenna</u> <u>Polarization</u>:

→ Antenna Polarization and the wave Polarization are identically same because the antenna radiates EM waves and also receives EM waves.

* Magnetic Vector Potential ($\bar{A}$).

→ A divergence less ~~Space~~ vector may be a curl of some other vector

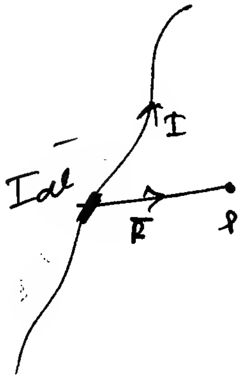
$$\nabla \cdot \bar{B} = 0$$

$$\therefore \quad \bar{B} = \nabla \times \bar{A}$$

$\bar{A}$ is named as magnetic vector potential.

$\bar{A}$: Magnetic Vector Potential

$\bar{A} \rightarrow$ T·m (or) wb/m.



$$d\bar{A} = \frac{\mu I d\bar{l}}{4\pi |R|}$$

$$\bar{A} = \int d\bar{A}$$

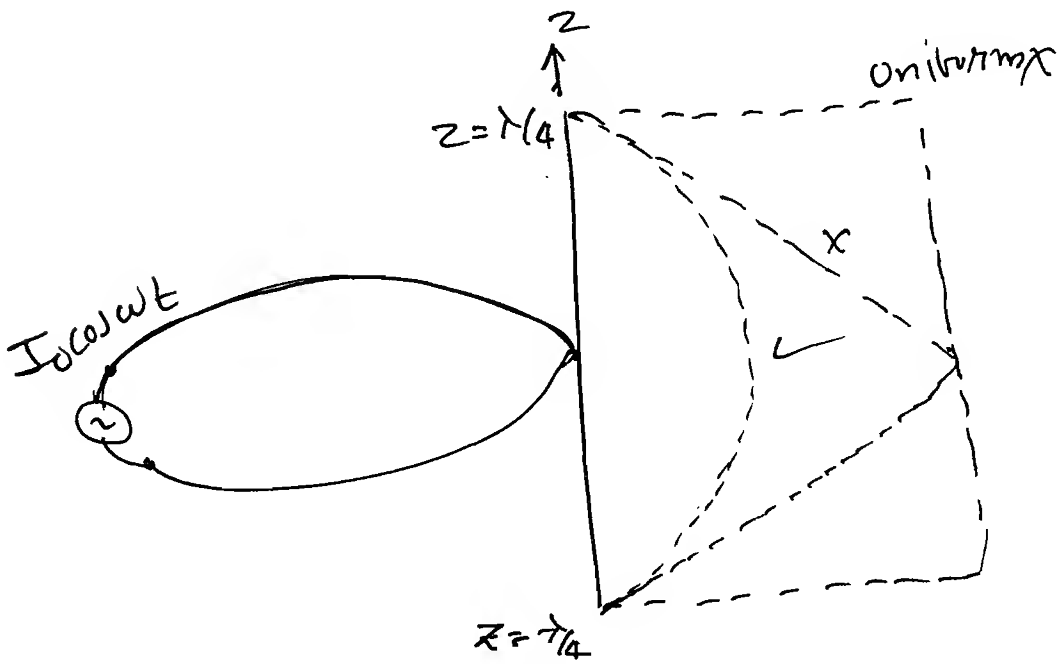
→ Source of magnetic potential is current element which is a vector quantity therefore ~~its~~ this magnetic potential is named as magnetic vector potential.

* Current distribution:

→ when an antenna is excited by a transmission line (or) by some means distribution of currents takes place on the antenna geometry. ~~as a function of~~

for e.g.

→ The suitable current distribution a half-wave dipole antenna is sinusoidal i.e. at a centre it is maximum and adjecent is minimum. This current disaribution is used in the process of antenna analysis



→ on a halfwave dipole antenna:

$$[I] = I_0 \cos\omega t \cdot \cos\beta z.$$

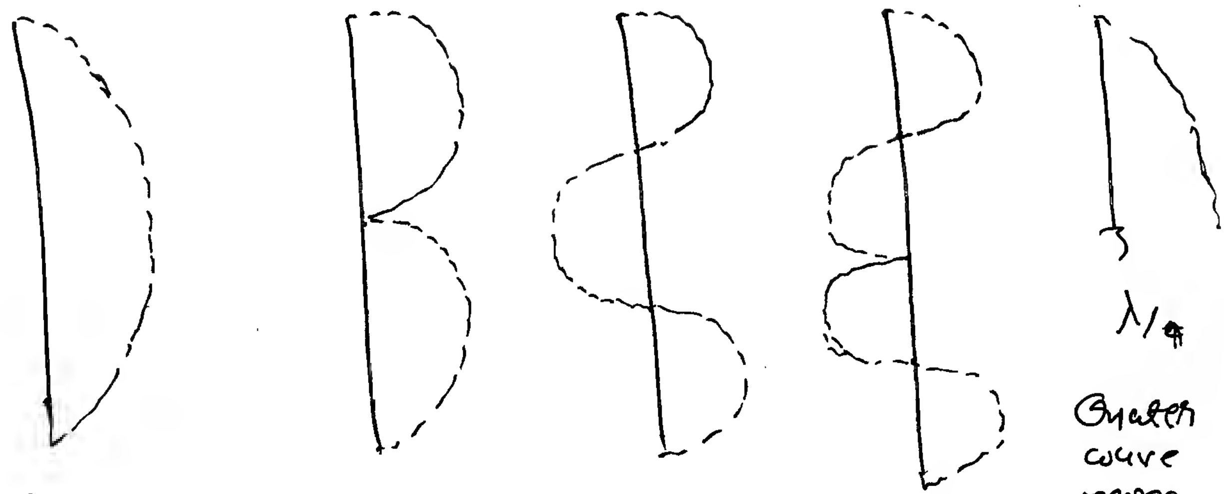
$$I_s = I_0 \cos\beta z.$$

at $z = \pm\lambda/4$, $\beta z = \pm\pi/2 \Rightarrow \cos\beta z = 0.$

$z = 0$, $\beta z = 0$, $\Rightarrow \cos\beta z = 1.$

* Current Distribution on the wire Antennas:

⇒



$\lambda/2$ | λ | 1.5λ | 2λ | $\lambda/4$

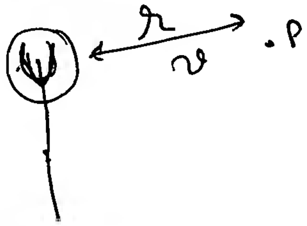
Half wave diopole | | | | Quater wave mono pole.

→ A Quatter wave monopole is backed by an infinite ground plane.

* Antenna analysis Procedure:

Step-1: Assume a suitable current distribution.

Step-2: Find $\bar{A}$ at a distant point



→ This potential is also named as retarted potential. It is retarded by r/v seconds.

Step-3: Find $\bar{H}$

$$\bar{B} = \nabla \times \bar{A}.$$

and $\bar{H} = \frac{1}{\mu} \nabla \times \bar{A}.$

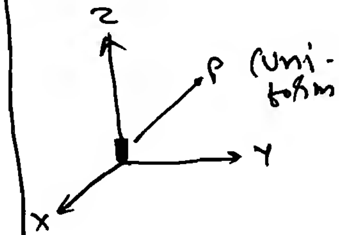
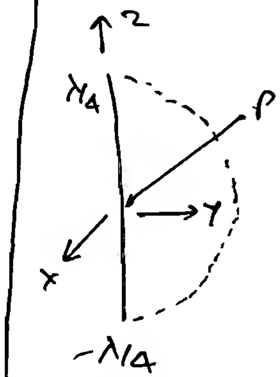
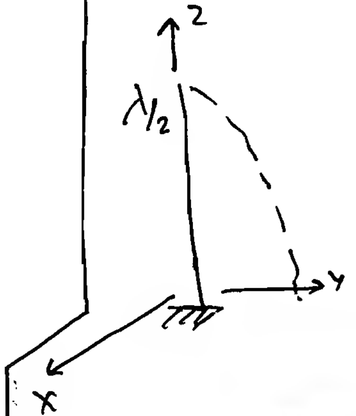
Step-4:

find $\bar{E}$.

$$\nabla \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t}.$$

(or) $$\bar{E} = \frac{1}{\epsilon} \int (\nabla \times \bar{H})\, dt.$$

Step-5: Investigate all the antenna parameter and there by one can decide the performance of an antenna.

Sr. No.	Name of the antenna	length (l)	Geometry & current distr.	$H_{\phi s}$	$E_{\theta s}$	W_{rad} (watts)	R_{rad} (Ω)	D_g	D_0
1	Hertzian dipole	$l \ll \lambda$		$\frac{jI_0 \beta l}{4\pi r} \sin\theta e^{-j\beta r}$	$\eta H_{\phi s}$	$80\pi^2 \left(\frac{l}{\lambda}\right)^2 \cdot I_{eff}^2$	$80\pi^2 \left(\frac{l}{\lambda}\right)^2$	$1.5 \sin^2\theta$	1.5
2.	Half wave dipole	$l = \lambda/2$	 (sinusoidal)	$\frac{jI_0 \cos(\frac{\pi}{2}\cos\theta)}{2\pi r \sin\theta} \cdot e^{-j\beta r}$	$\eta H_{\phi s}$	$73 I_{eff}^2$	73	$\frac{1.64\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$	1.64
3.	Quarter wave monopole	$l = \lambda/4$		$\frac{jI_0 \cos(\frac{\pi}{2}\cos\theta)}{2\pi r \sin\theta} \times e^{-j\beta r}$	$\eta H_{\phi s}$	$36.5 I_{eff}^2$	36.5	$\frac{3.28\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$	3.28

$I_{eff} = I_0/\sqrt{2}$

$\eta = \sqrt{\mu/\epsilon}$

→ Hetzian dipole also called infinitisimal dipole. It is an impractical antenna. It is used as a basic building block for analysing finite length antennas.

→ A Quarter wave Monopole is backed by an infinite ground plane. Therefore it radiates in the upper hemisphere only.

→ In the above analysis we have considered inverse distance terms only. other terms we have ignored.

→ The above radiated fields represents the waves which are propogating in the radial direction i.e. $\hat{a}_r$ direction.

$$H_{\phi_s} = \underbrace{\frac{jI_0\beta l}{4\pi r} \sin\theta}_{G} \, e^{-j\beta r}$$

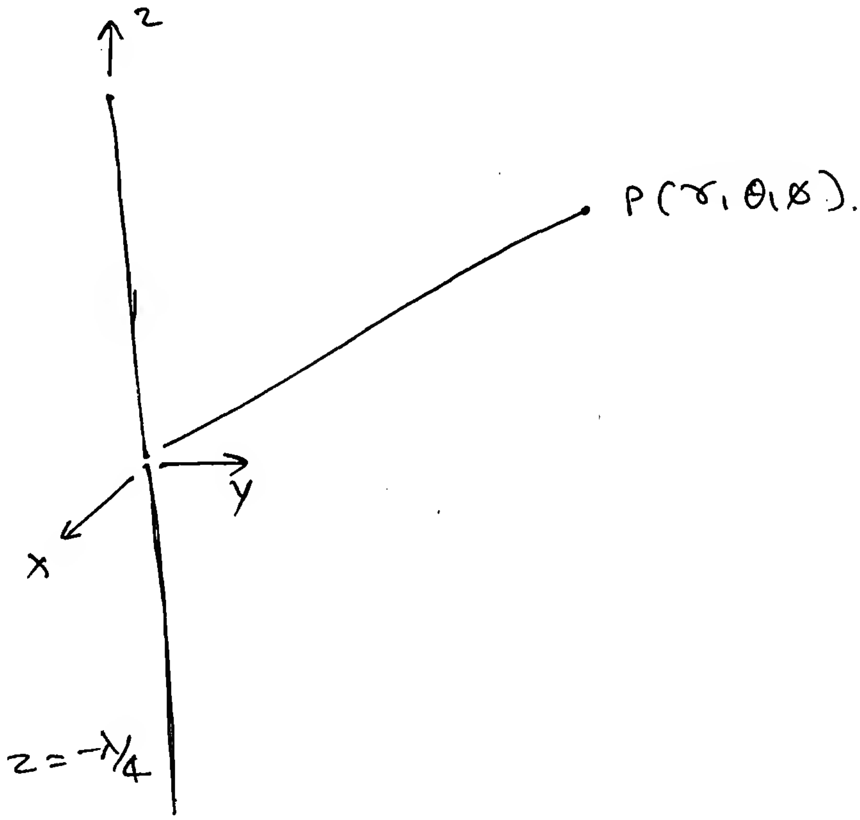
$$H_\phi = Re\left[G e^{j(\omega t - \beta r)}\right].$$

$$H_{\phi_s} = G e^{-j\beta r}$$

$\longrightarrow (r)$

$$\frac{E_\theta}{H_\phi} = \eta.$$

*



$$\rightarrow \quad E_{\theta r} = \left[\frac{j\eta I_0}{2\pi r} \cdot e^{-j\beta r}\right] \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}.$$

[fixed far distance r = const.].

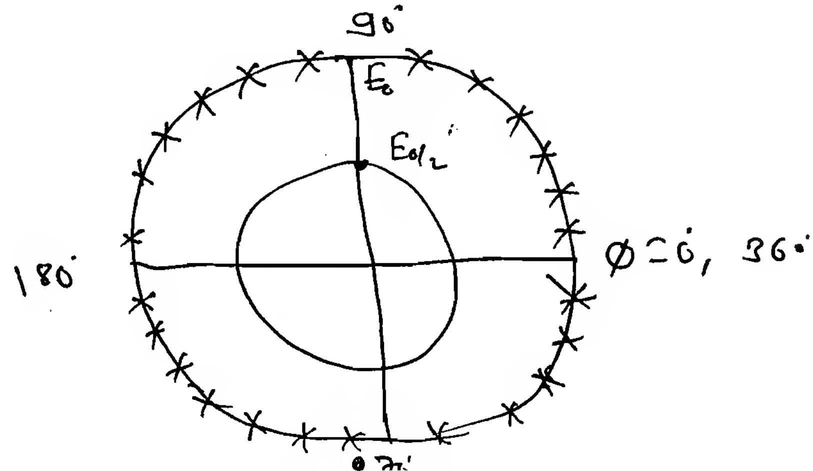
$$E_{\theta r} = E_0 \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}.$$

* Azimuthal plane pattern:

$\theta = 90°$, $\phi \rightarrow$ 0 to 360°.

$$|E_{\theta r}| = E_0 \cdot \left|\frac{\cos\left(\frac{\pi}{2}\cos 90°\right)}{\sin 90°}\right|$$

$$\therefore |E_{\theta r}| = E_0$$



→ Circular locus on the polar plot indicates that halfwave dipole antenna radiates uniformly in the azimuthal plane.

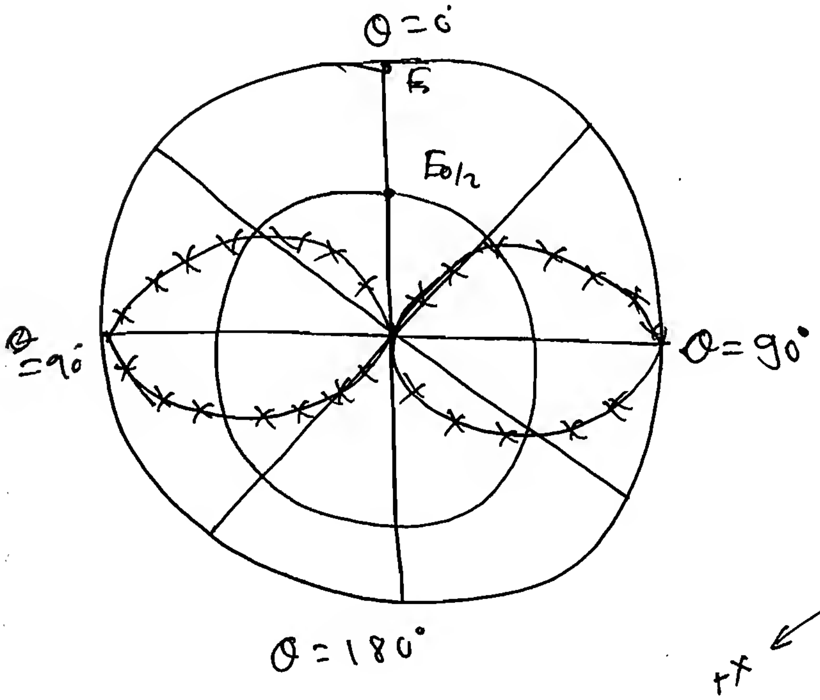
* Elevation plane: Pattern:

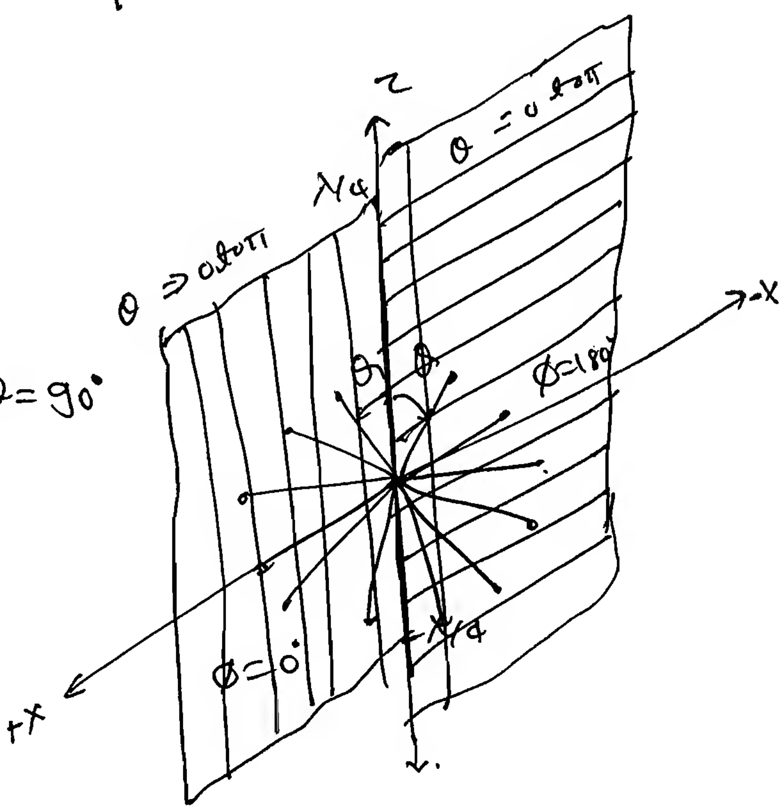
→ $\phi =$ const.

We consider $\phi = 0°$ and $\phi = 180°$.

$\theta \rightarrow 0$ to $180°$.

$$|\bar{E}_r| = \left| E_0 \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|$$





→ Locus on the Polar Plot indicates that in the elevation plane half wave dipole antenna radiates non-uniformly. Further we can also say that along the line of the antenna it has zero radiation and normal to the line of the antenna it has maximum radiations.

→ an antenna which is having uniform radiations in the azimuthal plane and having non-uniform radiation in the elivation plane than that kind of antenna is said to omnidirectional antenna. therefor dipole antenna is omnidirectional.

→ Quaterwave Monopole doesn't radiate for $90^\circ \le \theta \le 180^\circ$. because it is backed by an infinite ground plane.

Ex-1 It is reqired to absorve a magnetic field strength of $5 \mu A/m$ at $r = 2km$, $\theta = 90^\circ$. How much power that antenna will to radiate if it is a
(i) Hetzian dipole of length $\lambda/20$.
(ii) Halfwave dipole
(iii) Quarterwave Monopoles.

Ans: (1) Hetzian dipole

$l = \lambda/20$, $r = 2\times10^3 m$, $\theta = 90^\circ$

$|H_s| = |H_{\phi s}|$

$= 5\times10^{-6}$ A/m.

$$|H_{\phi s}| = \left| \frac{j I \beta l}{4\pi r} \sin\theta \cdot e^{-j\beta r} \right|$$

$$= \frac{I_0 \beta l}{4\pi r} \cdot \sin\theta \quad \Big| \quad \beta = \frac{2\pi}{\lambda}.$$

$\therefore$ 8 $I_0 =$

$\rightarrow$ $I_{eff} = I_0/\sqrt{2}$

$\rightarrow$ $W_{rad} = R_{rad} \cdot I_{eff}^2$

$$= 80\pi^2 \left(\frac{l}{\lambda}\right)^2 I_{eff}^2.$$

$\rightarrow$ ② $|H_s| = |H_{\phi s}| = \left| \frac{j I_0}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \cdot e^{-j\beta r} \right|$

$$= \frac{I_0}{2\pi r} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}.$$

$I_0 =$

$I_{eff} = I_0/\sqrt{2}.$

$\therefore$ $W_{rad} = I_{eff}^2 \cdot R_{rad} = I_{eff}^2 (73)$ watts

③ I_{eff} is same as halfwave dipole.

$W_{rad} = I_{eff}^2 (36.5)$ watts.

☆ Scattering Parameters:

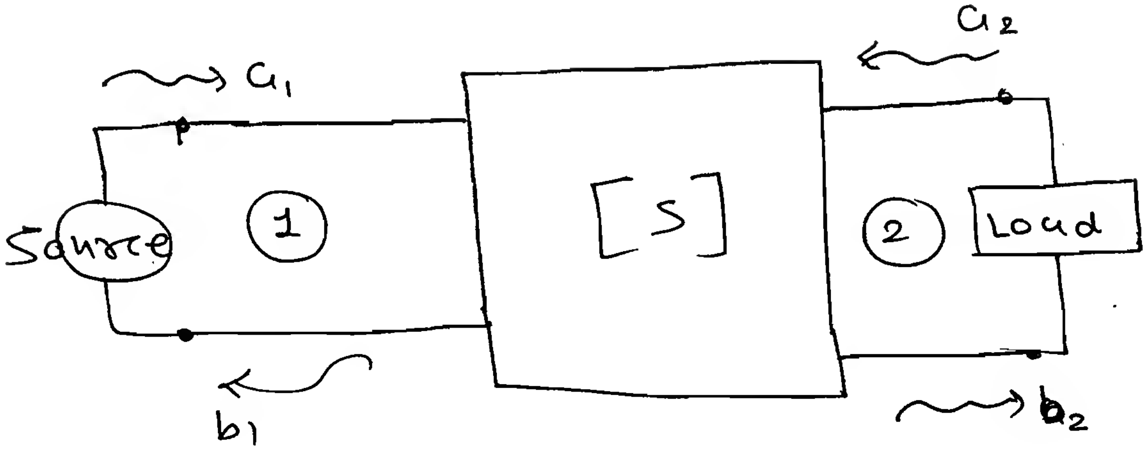
→ Conventional two port parameters such as z, y, h, T, g, and T' are less useful for the freq. beyond 1 GHz because of the following reasons.

① Equipment is not readily available for the measurment of port voltages and currents.

② Active devices like power Transistors, Tunnel Diodes becomes frequently unstable due to open and short circuit (conventional two port parameters are define as a ratio of V-V (OR) V-I (OR) (I-V) (OR) (I-I) and by open circuiting (OR) short circuits one of the ports)

→ ③ It is impractical to realized an ideal open circuit at microwave frequencies. Due to the above reasons S parameters ~~are~~ (OR) Scattering Parameters are used for analysing the circuits beyond 1 GHz.

* $\boxed{\text{S-parameters:}}$



→ All a_s are entering the port.

→ All b_s are leaving the port.

→ They are related by,

$$\boxed{\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2. \\ b_2 &= S_{21}a_1 + S_{22}a_2. \end{aligned}}$$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}.$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

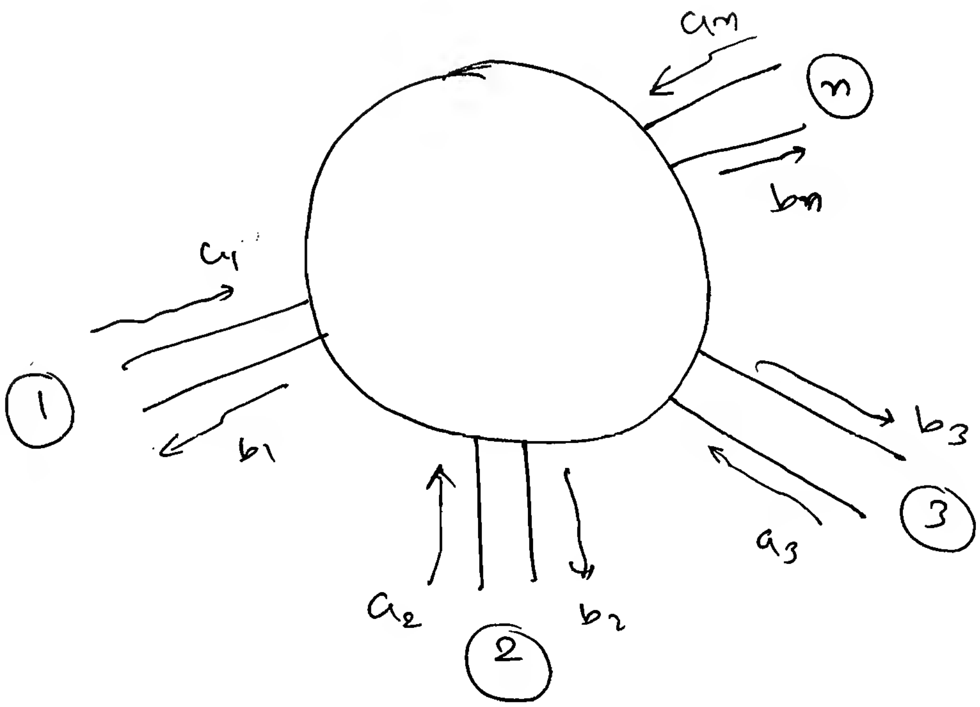
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_2=0}.$$

$\Rightarrow [b] = [S][a].$

$\Rightarrow [S]$: Scattering matrix

→ Coefficient in S are called Scattering coefficient.

→ In general microwave junction may have 'n' ports.



→ $b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + \cdots + S_{1n}a_n$

$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + \cdots + S_{2n}a_n$

$\vdots$

$b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \cdots + S_{nn}a_n$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2n} \\ \vdots & & & & \\ S_{n1} & S_{n2} & S_{n3} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

→ Reflection coefficient

→ In μwave junction will have $\underline{\underline{n^2}}$ scattering coefficient.

→ All scattering coefficients are dimensionless quantities

→ S_{ij} : Reflection coeff. of i^{th} port, if $i=j$, with all other ports are matched.

→ S_{ij} : Forward transmission coeff. of j^{th} port if $i>j$, with all other ports are matched.

→ S_{ij} : ~~Forw~~ Reverse transmission coeff. of j^{th} order if $i<j$, with all other ports are matched.

→ The main diagonal elements of S-matrix are reflection coefficients.

→ The elements below the main Diagonal ~~~~ ~~to~~ are forward transmission coefficient.

and

→ The elements above the main diagonal are Reverse Transmission coefficients.

* Properties of Scattering parameters:

① If all the main diagonal elements are zero then all the ports of the N/W are matched.

$$S_{11} = S_{22} = S_{33} = \cdots = S_{nn} = 0.$$

② Symmetry property

(or) $S_{ij} = S_{ji}$.

→ If the μwave junction do not have any non-reciprocal components ~~code~~ (or) active devices and the junction obeys reciprocity then $[S] = [S]^T$

③ Unity Property.

→ If the microwave junction is loss less then sum of the product of the complex conjugates of the elements in a Row (or) column is equal to unity.

→ $S_{11} S_{11}^* + S_{12} S_{12}^* = 1$. (Row 1)

→ $S_{21} S_{21}^* + S_{22} S_{22}^* = 1$ (Row 2).

→ $S_{11} S_{11}^* + S_{21} S_{21}^* = 1$ (column 1)

→ $S_{12} S_{12}^* + S_{22} . S_{22}^* = 1$ (column 2).

④ **Zero Property:**

→ Sum of the product of the complex conjugate of the elements of the other row (or) other column = 0. (Row to Row, column to column).

$$\therefore \quad S_{11} S_{21}^* + S_{12} S_{22}^* = 0 \quad \text{(Row 1 \& 2)}$$

$$S_{11} S_{12}^* + S_{21} \cdot S_{22}^* = 0. \quad \text{(Col 1 \& 2)}.$$

* **Relation b/w, [S], [Z], [Y]:**

→ $\boxed{[S] = [Z-U][Z+U]^{-1}}$

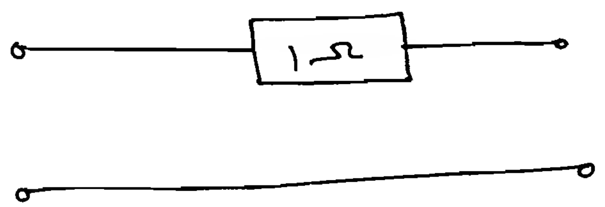
→ $\boxed{[S] = [U-Y][U+Y]^{-1}}$

→ $[Y] = [U] - 2[S][U+S]^{-1}$.

$[Z] = [U] + 2[S][U-S]^{-1}$.

Ex-1 Find [S] Parameters:

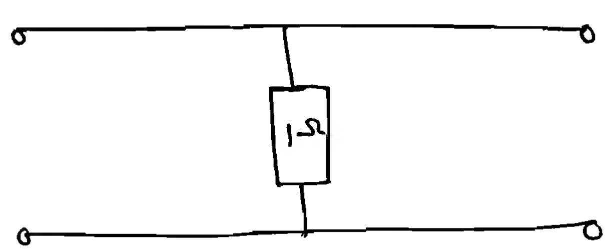
(a₁)



Step:1 find [Y]

Step:2 find [S]

(a₂)



Step 1: find [Z]

Step-2: [S].

Ex-2

$$[S] = \begin{bmatrix} 0.1\angle 40^\circ & 0.88\angle 30^\circ \\ 0.88\angle 30^\circ & 0.1\angle 40^\circ \end{bmatrix}$$

The N/w is

(a) Lossy and Reciprocical.

(b) Lossy and Non Reciprocal.

(c) Non-lossy and Reciprocal.

(d) Non-lossy and Non-Reciprocal

→ $[S] = [S]^T$ → Reciprocal.

$S_{11} \neq S_{12}$.

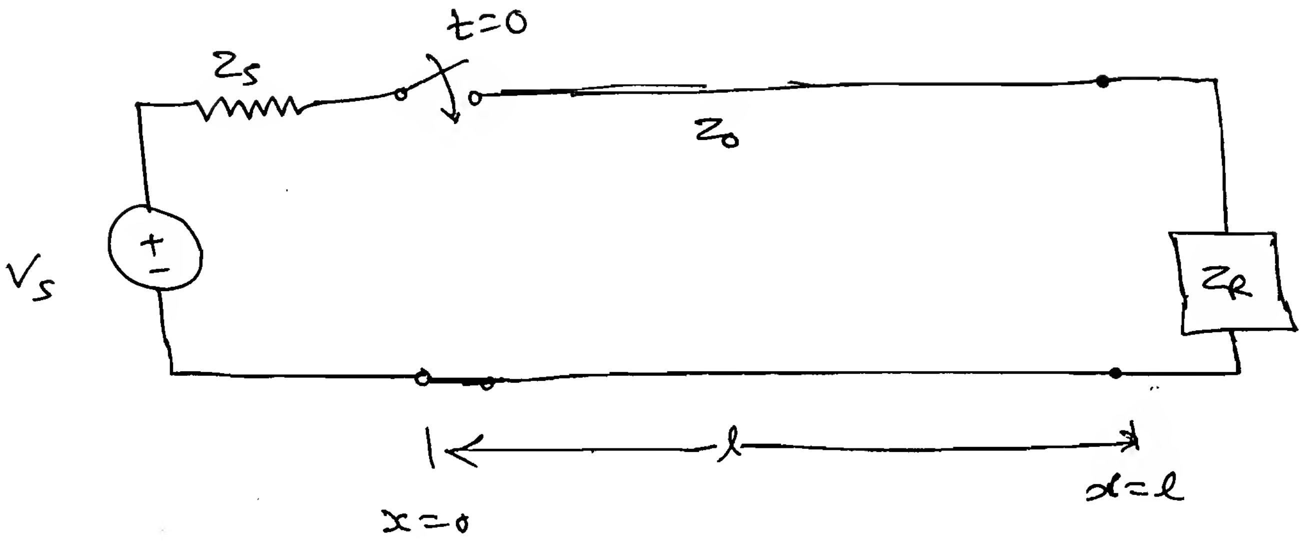
→ Ans: The above matrix satisfying symmetric condition therefore reciprocal and it is not satisfying unity property. Therefore it is lossy.

*

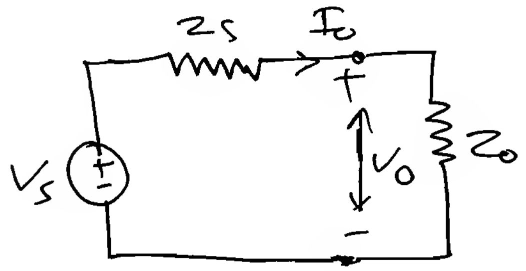
→ When a DC source is connected to a transmission line and is switched on it takes some time for the voltage and current on the line to reach a steady value. means Transition period is called Transient.

→ Fig. shows a transmission line driven by

a Dc Voltage Source.

→


→ Equavalanet ckt at $t=0^+$ & $x=0$,



→ at $t=0$, the switch is closed, then the storuge current 'sees' only Z_s and Z_0.

$$I\ (\text{at } x=0,\ t=0^+) = \frac{V_s}{Z_s+Z_0}.$$

$$\therefore\ V\ (\text{at } x=0,\ t=0^+) = \frac{Z_0}{Z_0+Z_s} V_s.$$

$$= I_0 Z_0$$

$$= V_0.$$

→ After the switch is closed $I_i = I_0$, $V_i = V_0$ propagates towards the load with a speed of $\frac{1}{\sqrt{LC}}$ m/s $(= v_p)$ (i = ~~consta~~ incident).

transient time $t_1 = \frac{l}{v_p}$.

[The wave would take certain time to reach the load].

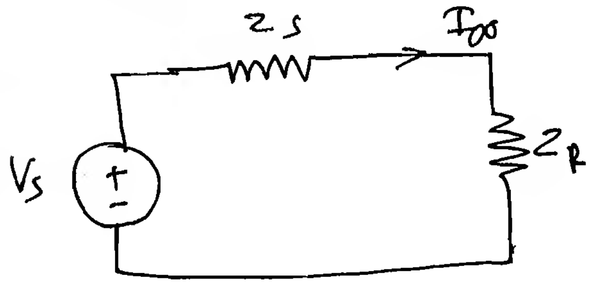
→ After 't_1' seconds the wave reach the load. The V and I at the load is the sum of incident and reflected.

$$V(l, t_1) = V_i + V_r = V_0 + kV_0.$$
$$= V_0(1+k).$$

$$\therefore k = \frac{Z_R - Z_0}{Z_R + Z_0}.$$

$$I(l, t_1) = I_i + I_r$$
$$= I_0 - kI_0$$
$$= I_0(1-k)$$

∴ Eqvt ckt at $t = \infty$



(I_∞: steady state current)

$$\therefore \boxed{I_\infty = \frac{V_s}{Z_s + Z_R}}.$$

Ex-1 Let $V_s = 30V$, $Z_s = 0$, $Z_0 = 50\,\Omega$.
$t_1 = 400\,\mu s$.

$V(at\ l, t_1 = 400\,\mu s) = 40V$.

Find k, Z_R and I_∞.

$\therefore$ V (at $x=l$, $t=400\mu s$) $= 40V = V_i + V_R$

$= V_0(1+K)$. $\left(\because \begin{array}{l} Z_s = 0 \\ V_0 = V_s \end{array}\right)$.

$\therefore$ $40 = 30(1+K)$.

$\therefore$ $\boxed{K = 0.33.}$

$\therefore$ ~~K = 0.67~~.

$\rightarrow$ $\dfrac{Z_R - Z_0}{Z_R + Z_0} = K$.

$\therefore$ $\dfrac{Z_R}{Z_0} = \dfrac{1+K}{1-K}$.

$\therefore$ $Z_R = Z_0(2) = 100\,\Omega$.

$\therefore$ $I_\infty = \dfrac{V_s}{Z_s + Z_R} = \dfrac{30}{100}$.

$\therefore$ $\boxed{I_\infty = 0.3\,A}$

Ex: A Transmission line is terminated by a load impedence of $50 + 50j\ \Omega$. Assume characteristics impedence of the line is $50\ \Omega$.

(i) Locate Normalized load impedence on the smith chart.

Ans:

$Z_R = 50 + j50$

$Z_0 = 50$

$z_r = \frac{Z_R}{Z_0}$.

$\therefore \boxed{z_r = 1 + j1}$

→ Point A on the Smith chart indicates Normalized load impedence.

(ii) Using Smith chart find normalized admittance.

$$y_r = \frac{1}{z_r} = \frac{1}{1-j} = \frac{1-j}{(\sqrt{1+1})^2} = \frac{1}{\sqrt{2}} \; 0.5 - 0.5j$$

→ The value at B indicates normalized load admittance $\boxed{y_r = 0.5 + j0.5}$.

(iii) find VSWR:

→ The value at C and value at D also indicates VSWR and the value is 2.6.

(iv) Find max. voltage reflection coefficient

$$|K| = \frac{S-1}{S+1} = \frac{2.6-1}{2.6+1}.$$

$|\Gamma| = 0.44.$

The value at E represent mag. of the reflection coefficient and is of 0.45

(V) Also find phase angle of the reflection coefficient.

Ans: The value at F represent the phase angle of the voltage reflection coefficient and value is 65°.

(vi) Power reflection coefficient = ?

Ans: The value at G indicates the power reflection coefficient and the value is 0.2.

(vii) Find maximum impedence and the minimum impedence on the transmission line?

Ans. The value at D indicates normalized

$$\frac{Z_{max}}{Z_0} =$$

$$Z_{max} = SZ_0. \qquad Z_{min} = Z_0/S.$$

$$\frac{Z_{max}}{Z_0} = S.$$

→ the value at D indicates normalized maximum impedence on the transmission line and the value at H indicated minimum impedence on the transmission line.

$\therefore$ $Z_{max} = 2.6 \times 50.$

$Z_{min} = \frac{50}{0.4}.$

(viii) Find impedence on the transmission line at a distance of 0.1λ from the load.

Ans: The value at J indicated normalized impedence and the value is $z_r = 2.6 - j0.4.$

$\therefore$ $Z = (2.6 - j0.4)\, 50\, \Omega$

(10) Find the impedence on the transmission line at distance. ~~$\lambda = 0.34\lambda$~~ $\lambda = 0.5$ from Load.

→ The point A it self is the impedence at a distance of 0.5λ from the load. which is load impidence.

(11) Find the impedence on the transmission line at a distance of 0.25λ from the load.

Ans: The point B indicates the answer of the quetion.

arc line KC is 0.25λ.

→ The point B indicates normalized load impedence at a distance 0.25λ from the load.

NOTE: With O as centre OA as radius we drawn a circle. That circle indicates Locus of the impedence on the transmission line.

(11) Find the distance betn load and the first voltage max.

→ ~~Ar~~ line k m indicates the distance betn the load and the first voltage max.

$0.25\lambda - 0.16\lambda = 0.09\lambda$

(12) Find the distance betn load and first voltage minimum..

$0.5 - 0.16 = 0.34.\lambda$

∴

(or) $0.09 + 0.25 = 0.34\lambda.$

The ~~kok~~ line k indicates the distance betn the load and first voltage minimum the ~~Vam~~ distance is 0.34λ

(13) how many maximas and ~~how~~ many minimas occur on the transmission line it the line length is 2.02λ.

Ans:

length	cumulative length	V_{max}	V_{min}
0.5λ	0.5	1	1
0.5λ	1.0λ	1	1
0.5λ	1.5λ	1	1
0.5λ	2.0λ	1	1
0.02λ	2.02λ	—	—
		4	4

4-maxima & 4-minima.